PENDAHULUAN

Materi kuliah hingga UTS :

Minggu ke-	Kemampuan Akhir Sesuai Tahapan Belajar (Sub-CPMK)	Materi Pembelajaran
(1)	(2)	(3)
1	Sub-CPMK 1 : Pendahuluan & Pengenalan Sistem Telekomunikasi **[CLO 1]**	1. Pengenalan Silabus, sasaran pengajaran, referensi
		2. Kontrak belajar dan Aturan penilaian: Quis, Ujian, Tugas dll
		3. Pengenalan sistem dan blok sistem telekomunikasi (ulas ulang)
		4. Perkembangan Sistem Telekomunikasi
		5. Review Parameter Telekomunikasi (tegangan, Arus, Daya, Energi, Bandwidth)
	Sub-CPMK 2 : Transformasi Fourier **[CLO 1]**	1. Pemahaman dan arti penting domain waktu dan domain frekuensi
		2. Review deret Fourier, transformasi Fourier
		3. Contoh transform sinyal rectangular
		4. Sifat-sifat Transformasi Fourier
		5. Contoh transformasi Fourier dan aplikasi sifat-sifatnya
2,3,4	Sub-CPMK-3 : Sistem AM **[CLO 2]**	1. Pemahaman arti dan fungsi Modulasi dan Demodulasi
		2. Modulator AM-DSB-SC : Modulator dan Demodulator (Blok, persamaan), Gambar spektral, bandwidth, perhitungan daya
		3. Konsep translasi frekuensi
		4. AM-SSB : Modulator-demodulator, Gambar spektral, bandwidth, perhitungan daya
		5. AM-DSB-FC : Modulator-demodulator, persamaan, indeks modulasi, konstanta modulasi, Detektor selubung, Gambar spektral, bandwidth, perhitungan daya
	Sub-CPMK-4 : Sistem FM **[CLO 2]**	1. Modulator FM : Persamaan, indeks modulasi, fungsi Bessel, Spektral, Daya, BW, blok sistem
		2. Demodulator FM : Persamaan, blok sistem
		3. Superheterodyne pada FM

5	Sub-CPMK-5 : Noise pada Sistem Telekomunikasi **[CLO 2]**	1. Jenis-jenis noise dalam sistem komunikasi
		2. AWGN : sifat, persamaan
		3. Gambaran distribusi noise yang bukan AWGN (mis : uniform)
	Sub-CPMK-6 : Sistem Pradeteksi dan Kinerja Pradeteksi **[CLO 2]**	1. Struktur rangkaian pradeteksi dan blok penyusun
		2. Parameter rangkaian pradeteksi : Gain, Redaman, Temperatur Noise ekuivalen, Rapat spektral daya noise, daya noise, BW
		3. Kinerja rangkaian Pradeteksi
		4. Sistem Cascade, parameter cascade, perhitungan kinerja dalam bentuk Cascade
6	Sub-CPMK-7 : Kinerja AM **[CLO 2]**	1. Kinerja AM-DSB-SC
		2. Kinerja AM-SSB
		3. Kinerja AM-DSB-FC
		4. Kinerja Sistem Modulasi AM (digabung dengan rangkaian pradeteksi)
7	Sub-CPMK-8 : Kinerja FM **[CLO 2]**	1. Kinerja Modulasi FM
		2. Figure of Merit
		3. Kinerja Sistem Modulasi FM (digabung dengan rangkaian pradeteksi)

Text Book :

1). Comm Sys , Bruce Carlson , 5th ed

2). Digital Communication for Practicing Engineer 2020 - 1th ed

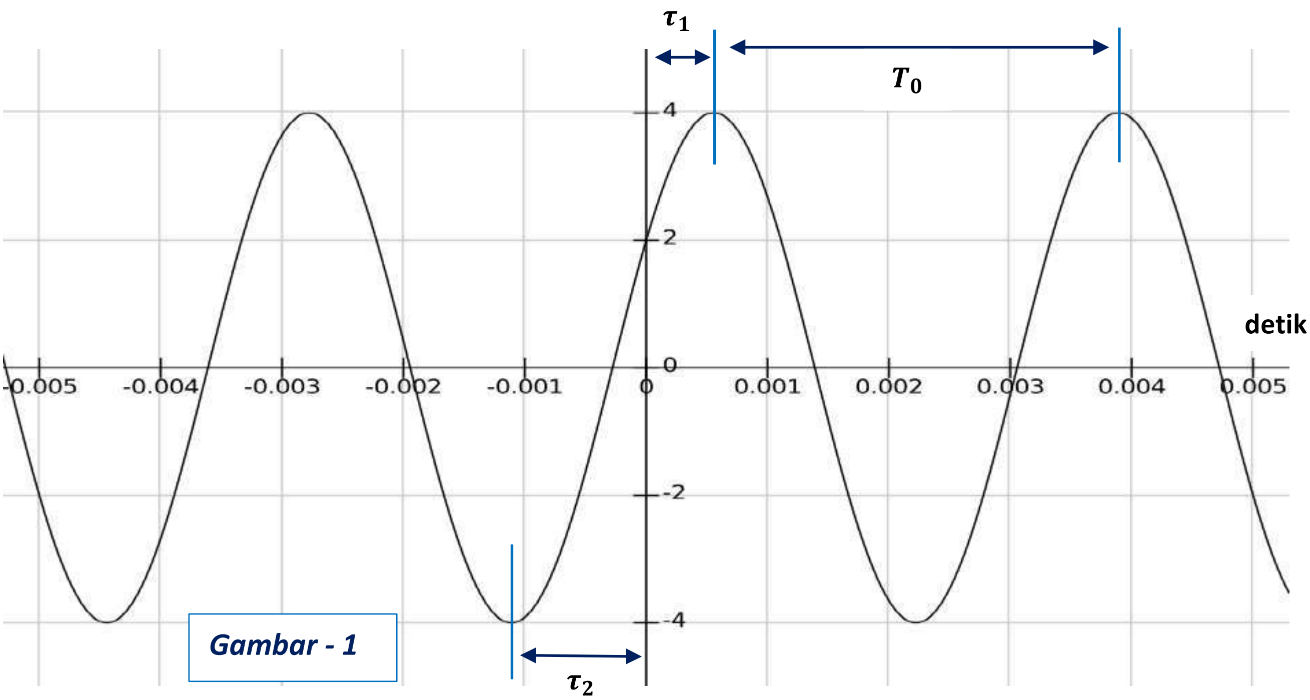
Penilaian :

NILAI AKHIR = 35% UTS + 35 % UAS + 30 % (Tugas + Kuis)

Sinyal dalam banyak text Book sistem komunikasi maka ***maksudnya sinyal tegangan***

Misal disebutkan ***sinyal x(t)*** serta tak ada keterangan tambahan maka yang dimaksud adalah ***sinyal tegangan dengan satuan*** $Volt$, kecuali ada penjelasan tambahan yang menyertainya .

Sinyal Sinussoidal



Gambar - 1

$$frekuensi \ = \ f = \frac{1}{T_0}$$

$$Sinyal \ \ pd \ gambar \ di \ atas \ dapat \ dituliskan \ \ : \ \ x(t) = 4\sin(2\,\pi \times 300\,t - \ 30^0) \ Volt$$

$$atau \ : \ \ x(t) = 4\sin(2\,\pi \times 300\,t - \mathbf{0,5236}) \ Volt$$

Perhatikan bahwa : $30^0 \ \ (\, 30 \, derajat \,) = \frac{30}{180} \times \pi \ \ rad = \ \ 0{,}5236 \ rad$

$$Dapat \ juga \ dituliskan \ \ : \ \ x(t) = 4\cos(2\,\pi \times 300\,t + \ 60^0) \ Volt$$

$$atau \ : \ \ x(t) = 4\sin(2\,\pi \times 300\,t + \mathbf{1,0472}) \ Volt$$

Sinyal dalam format ***sinus*** selalu dapat dinyatakan dalam format ***cosinus***

Bila dalam literatur atau text book disebutkan ***sinyal sinussoidal*** maka yang dimaksud adalah sinyal dalam format sinus atau cosinus

$$\boldsymbol{x(t) = \ 4 \ \ \sin(2\,\pi \times \ \ 300 \ \ t \ \ - 30^0) \ \ Volt}$$

Sinyal tersebut memiliki amplituda = 4 Volt   ,   frekuensi = 300 Hz

Bentuk umum sinyal sinusoidal frekuensi tunggal :

$$s(t) = A\ sin(2\,\pi\,f\,t\ +\ \theta) \quad \text{atau} \quad s(t) = A\ \cos(2\,\pi\,f\,t\ +\ \emptyset)$$

Energi , Daya rata-rata , Daya puncak

Misal suatu sinyal ***x(t)*** pada beban resistif ***R***

$$Energi\ sinyal\ x(t) = E_s = \frac{1}{R}\int_{-\infty}^{+\infty} [x(t)]^2$$

$$Daya\ \ rata-rata\ sinyal\ x(t) = P_{av} = \frac{1}{T\,R}\int_{-\frac{T}{2}}^{+\frac{T}{2}} [x(t)]^2$$

Pada bahasan sistem komunikasi bila nilai beban R tak disebutkan berarti diasumsikan R = 1 Ohm sehingga :

$$Energi\ sinyal\ x(t) = E_s = \int_{-\infty}^{+\infty} [x(t)]^2$$

$$Daya\ rata-rata\ sinyal\ x(t) = P_{av} = \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} [x(t)]^2$$

Misalkan ***sinyal sinussoidal* s(t)** tersebut pada beban resistif murni sebesar ***R*** Ohm

Daya rata-rata sinyal sinusoidal pada beban ***R*** = $P_{av} = \frac{A^2}{2R}$ watt

Daya puncak sinyal sinusoidal pada beban ***R*** = $P_{peak} = \frac{A^2}{R}$ watt

Bila nilai R tak disebutkan maka asumsikan R = 1 Ohm , sehingga :

$$P_{av} = \frac{A^2}{2} \text{ watt} \qquad \text{dan} \qquad P_{peak} = A^2 \text{ watt}$$

Misalkan suatu sinyal dc sebesar $\boldsymbol{A}$ **Volt**

Daya rata-rata sinyal dc tsb pada beban ***R*** = daya puncaknya $P = \frac{A^2}{R}$ watt

Bila nilai R tak disebutkan maka asumsikan R = 1 Ohm , sehingga : $P = A^2$ watt

Satuan dB

Rumus konversi dari bilangan real kedalam dB :

$$\boldsymbol{B\ kali} = \mathbf{10} \times {}^{\mathbf{10}}\mathbf{log}(\boldsymbol{B}) \quad \boldsymbol{dB}$$

Contoh :

$\mathbf{1}\ \boldsymbol{kali} = 10 \times {}^{10}\log(1) = 10 \times 0 = \mathbf{0}\ \boldsymbol{dB}$

$100\ kali = 10 \times {}^{10}\log(100) = 10 \times 2 = 20\ dB$

$1000\ kali = 10 \times {}^{10}\log(100) = 10 \times 3 = 30\ dB$

$2\ kali = 10 \times {}^{10}\log(2) \approx 10 \times 0{,}30103 = 3{,}0103\ dB \approx 3\ dB \quad (\,pembulatan\,)$

$4\ kali = 10 \times {}^{10}\log(4) \approx 6\ dB \quad (\,pembulatan\,)$

$8\ kali = 10 \times {}^{10}\log(8) \approx 9\ dB \quad (\,pembulatan\,)$

$\frac{1}{2}\ kali = 10 \times {}^{10}\log\left(\frac{1}{2}\right) \approx -3{,}0103\ dB \approx -3\ dB \quad (\,pembulatan\,)$

$\frac{1}{4}\ kali = 10 \times {}^{10}\log\left(\frac{1}{4}\right) \approx -6{,}0201\ dB \approx -6\ dB \quad (\,pembulatan\,)$

$\frac{1}{8}\ kali = 10 \times {}^{10}\log\left(\frac{1}{8}\right) \approx -9{,}031\ dB \approx -9\ dB \quad (\,pembulatan\,)$

$\mathbf{2000}\ kali = 10 \times {}^{10}\log(2 \times 1000) = 10 \times {}^{10}\log(2) + 10 \times {}^{10}\log(1000) =$

$$\approx \mathbf{3}\ \boldsymbol{dB} + \mathbf{30}\ \boldsymbol{dB} = \mathbf{33}\ \boldsymbol{dB}$$

$\mathbf{5000}\ kali = 10 \times {}^{10}\log\left(\frac{1}{2} \times 10000\right) = 10 \times {}^{10}\log\left(\frac{1}{2}\right) + 10 \times {}^{10}\log(10000) =$

$$\approx -\mathbf{3}\ \boldsymbol{dB} + \mathbf{40}\ \boldsymbol{dB} = \mathbf{37}\ \boldsymbol{dB}$$

$\mathbf{4000}\ kali = 10 \times {}^{10}\log(2 \times 2 \times 10000) = 10 \times {}^{10}\log(2) + 10 \times {}^{10}\log(10000) =$

$$\approx -\mathbf{3}\ \boldsymbol{dB} + \mathbf{40}\ \boldsymbol{dB} = \mathbf{37}\ \boldsymbol{dB}$$

Satuan dBm (dB mWatt) , dBW (dB Watt)

Rumus konversi :

$$\boldsymbol{B \ \ watt \ = \ 10 \times {}^{10}\log(B) \quad dB\,W}$$

$$\boldsymbol{X \ \ mWatt \ = \ 10 \times {}^{10}\log(X) \quad dB\,m}$$

$$\boldsymbol{C \ \ dBW = \ (\, C + 30 \,) \ \ dBm}$$

Contoh :

$\mathbf{1\ \mathit{Watt}} = 10 \times {}^{10}\log(1) = 10 \times 0 = \mathbf{0\ \mathit{dBW}}$

$100\ W = 10 \times {}^{10}\log(100) = 10 \times 2 = 20\ dBW$

$20\ W = 10 \times {}^{10}\log(20) \approx 13\ dBW$

$0{,}25\ W = 10 \times {}^{10}\log(0{,}25) \approx -6\ dBW$

$0{,}2\ W = 10 \times {}^{10}\log(0{,}2) \approx -7\ dBW$

$20\ W = 10 \times {}^{10}\log(20) \approx 13\ dBW = (\,13 + 30\,) = 43\ dBm$

$0{,}2\ W = 10 \times {}^{10}\log(0{,}2) \approx -7\ dBW = (\,-7 + 30\,) = 23\ dBm$

$0{,}2\ W = 200\ mW = 10 \times {}^{10}\log(200) \approx 23\ dBm$

$10^{-9}\ Watt = -90\ dBW = (-90 + 30\,)dBm = -60\ dBm$

Suatu sinyal dengan daya $P_i = -70\ dBm$ diperkuat oleh Amplifier dengan Gain sebesar 13 dB maka daya sinyal dioutput Amplifier :

$$P_o = (-70 + 13\,)\ dBm = -57\ dBm$$

$$-57\ dBm = (-60 + 3\,)\ dBm = 10^{-6}\ mWatt \times 2 = 2 \times 10^{-6}\ mWatt$$

$$-53\ dBm = (-50 - 3\,)\ dBm = 10^{-6}\ mWatt \times 0{,}5 = 0{,}5 \times 10^{-6}\ mWatt$$

Bahan diskusi :

1) Apa yang dimaksud 13 dB , apa bedanya dengan 13 dBm
2) Mana yang benar : Faktor penguatan = 20 dBW , Faktor Penguatan = 20 kali , Faktor penguatan = 20 dB , daya sinyal = $4 \times 10^{-6}\ Watt$, daya = 7 dBm , Daya sinyal = - 5 dBm , daya sinyal = - 5 Watt , daya sinyal = 5 dB

1. DERET FOURRIER

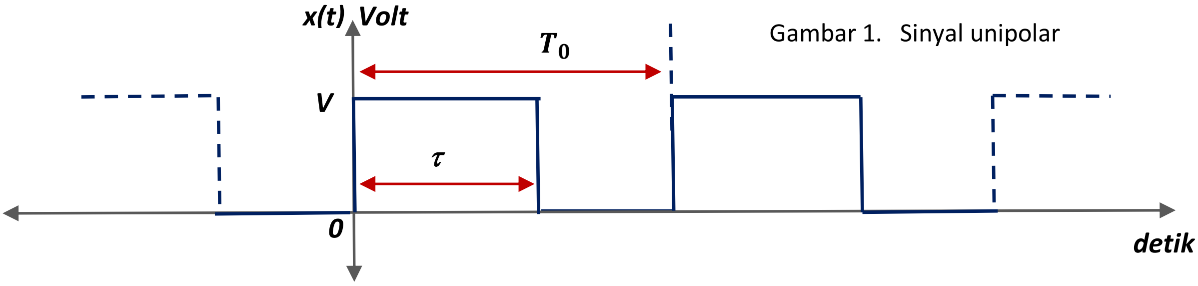
Tiap sinyal periodik x(t) dapat dinyatakan dalam bentuk deret sinyal sinusoidal .

$$\boldsymbol{x(t)} = a_0 + a_1 \cos(\omega_0 t) + a_2 \cos(2\omega_0 t) + a_3 \cos(3\omega_0 t) + \; . \; . \; . \; .$$

$$+ b_1 \sin(\omega_0 t) + b_2 \sin(2\omega_0 t) + b_3 \sin(3\omega_0 t) + \; . \; . \; . \; . \qquad ; \qquad \boldsymbol{\omega_0 = 2\pi f_0}$$

$$\boldsymbol{x(t)} = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

Contoh 1 : Mendapatkan deret Fourrier sinyal periodic segiempat Unipolar



Gambar 1. Sinyal unipolar

a). Menghitung koefisien Cosinus yaitu : $\boldsymbol{a_0 \; , \; a_1 \; , \; a_2 \; , \; . \; . \; . \; a_n}$

$$\boldsymbol{a_0} = \frac{\boldsymbol{1}}{\boldsymbol{T_0}} \int_{t_x}^{t_x+T_0} \boldsymbol{x(t)} \; \boldsymbol{dt} \qquad ; \; \textit{lihat gambar} \; \rightarrow \boldsymbol{T_0} \text{ adalah nilai perioda}$$

$$\boldsymbol{a_1} = \frac{\boldsymbol{2}}{\boldsymbol{T_0}} \int_{t_x}^{t_x+T_0} \boldsymbol{x(t)} \cos(\omega_0 t) \; \boldsymbol{dt} \qquad ; \quad \boldsymbol{f_0} = \frac{\boldsymbol{1}}{\boldsymbol{T_0}} \; , \quad \boldsymbol{\omega_0 = 2\pi \; f_0}$$

$$\boldsymbol{a_2} = \frac{\boldsymbol{2}}{\boldsymbol{T_0}} \int_{t_x}^{t_x+T_0} \boldsymbol{x(t)} \cos(2\,\omega_0 t) \; \boldsymbol{dt} \qquad ; \qquad \boldsymbol{a_n} = \frac{\boldsymbol{2}}{\boldsymbol{T_0}} \int_{t_x}^{t_x+T_0} \boldsymbol{x(t)} \cos(n\,\omega_0 t) \; \boldsymbol{dt}$$

Nilai $\boldsymbol{t_x}$ dapat dipilih sembarang , jadi biasanya dipilih yang memudahkan perhitungan integral

Pada contoh ini dipilih $\boldsymbol{t_x = 0}$ sehingga : (lihat gambar)

$$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t) \; dt = \frac{1}{T_0} \int_0^{\tau} V \; dt = \frac{V}{T_0} \; t \Big|_{t=0}^{t=\tau} = \frac{\boldsymbol{V}}{\boldsymbol{T_0}} \; \boldsymbol{\tau}$$

$$\boldsymbol{a_n} = \frac{\boldsymbol{2}}{\boldsymbol{T_0}} \int_0^{\tau} \boldsymbol{V} \cos(\boldsymbol{n\omega_0 t}) \; \boldsymbol{dt} = \frac{\boldsymbol{2V}}{\boldsymbol{n\,\omega_0\,T_0}} \sin(\boldsymbol{n\omega_0 t}) \Big|_0^{\tau} = \frac{\boldsymbol{2V}}{\boldsymbol{n\,\omega_0\,T_0}} \sin(\boldsymbol{n\omega_0\,\tau}) \quad ; \quad \boldsymbol{n > 0}$$

b). Menghitung koefisien Sinus yaitu : $\boldsymbol{b_1 \; , \; b_2 \; , \; b_3 \; , \; . \; . \; . \; b_n}$

$$\boldsymbol{b_n} = \frac{\boldsymbol{2}}{\boldsymbol{T_0}} \int_{t_x}^{t_x+T_0} \boldsymbol{x(t)} \sin(n\,\omega_0 t) \; \boldsymbol{dt} \qquad ; \; \textit{dipilih} \; \boldsymbol{t_x = 0} \quad , \qquad \textit{maka} :$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\ \sin(n\,\omega_0 t)\ dt = \frac{2}{T_0}\int_0^{\tau} x(t)\ \sin(n\,\omega_0 t)\ dt \quad ...(\textit{lihat gambar})$$

$$b_n = -\frac{2V}{n\omega_0\ T_0}\ \cos(\omega_0 t)\Big]_0^{\tau} = -\frac{2V}{n\,\omega_0\ T_0}\,[\cos(n\omega_0\,\tau) - \cos(0)]$$

$$b_n = \frac{2V}{n\,\omega_0\ T_0}\,[1 - \cos(n\omega_0\,\tau)\,] \quad ;\ n > 0$$

Dari hasil perhitungan di atas maka sinyal segiempat pada gb.1 dapat dituliskan dalam bentuk :

$$x(t) = \frac{V}{T_0}\,\tau + \sum_{n=1}^{\infty}\frac{2V}{n\,\omega_0\ T_0}\sin(n\omega_0\,\tau)\ \cos(n\omega_0 t\,)$$

$$+ \sum_{n=1}^{\infty}\frac{2V}{n\,\omega_0\ T_0}\,[1 - \cos(n\omega_0\,\tau)\,]\ \sin(n\omega_0 t\,)$$

$$\omega_0\,\tau = 2\pi\,f_0\,\tau = 2\pi\frac{\tau}{T_0} \qquad ;\ \omega_0\ T_0 = 2\pi\,f_0\,T_0 = 2\pi\,\frac{1}{T_0}\,T_0 = 2\pi$$

$$x(t) = \frac{V}{T_0}\,\tau + \frac{V}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(n\,2\pi\,\frac{\tau}{T_0}\right)\ \cos(n\omega_0 t\,)$$

$$+ \frac{V}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[1 - \cos\left(n\,2\pi\,\frac{\tau}{T_0}\right)\right]\sin(n\omega_0 t\,)$$

$$a_0 = \frac{V}{T_0}\,\tau \quad ;\ a_n = \frac{V}{\pi\,n}\sin\left(n\,2\pi\,\frac{\tau}{T_0}\right) \quad ;\ b_n = \frac{V}{\pi\,n}\left[1 - \cos\left(n\,2\pi\,\frac{\tau}{T_0}\right)\right]$$

Bentuk : $x(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\omega_0 t\,) + b_n\sin\,(n\omega_0 t\,)$, ***dapat dituliskan sbb*** :

$$x(t) = a_0 + \sum_{n=1}^{\infty} d_n\cos(n\omega_0 t - \theta_n) \quad ,\ d_n = \sqrt{(a_n)^2 + (b_n)^2}$$

Beberapa literatur menuliskan symbol d_n dengan c_n

Pada bahasan disini symbol c_n digunakan untuk Deret Fourrier bentuk Eksponensial

$$(a_n)^2 + (b_n)^2 = V^2\left(\frac{1}{\pi\,n}\right)^2\left(\sin\left(n\,2\pi\,\frac{\tau}{T_0}\right)\right)^2 + V^2\left(\frac{1}{2\pi\,n}\right)^2\left(1 - \cos\left(n\,2\pi\,\frac{\tau}{T_0}\right)\right)^2$$

$$d_n = \frac{V}{\pi\,n}\sqrt{\left(\sin\left(n\,2\pi\,\frac{\tau}{T_0}\right)\right)^2 + \left(1 - \cos\left(n\,2\pi\,\frac{\tau}{T_0}\right)\right)^2}$$

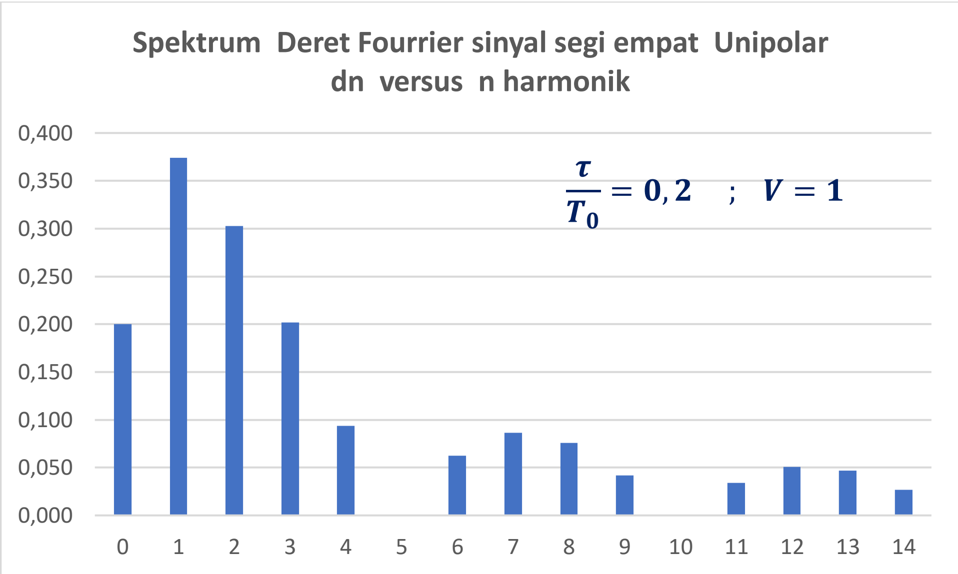
$$d_n = \frac{V}{\pi\, n} \sqrt{\left(\sin\left(n\, 2\pi \, \frac{\tau}{T_0} \right) \right)^2 + \left(\cos\left(n\, 2\pi \, \frac{\tau}{T_0} \right) \right)^2 + 1 - 2\, \cos\left(n\, 2\pi \, \frac{\tau}{T_0} \right)}$$

$$d_n = \frac{V}{\pi\, n} \sqrt{2 - 2\, \cos\left(n\, 2\pi \, \frac{\tau}{T_0} \right)} \quad ; \quad a_0 = \frac{V}{T_0}\, \tau$$

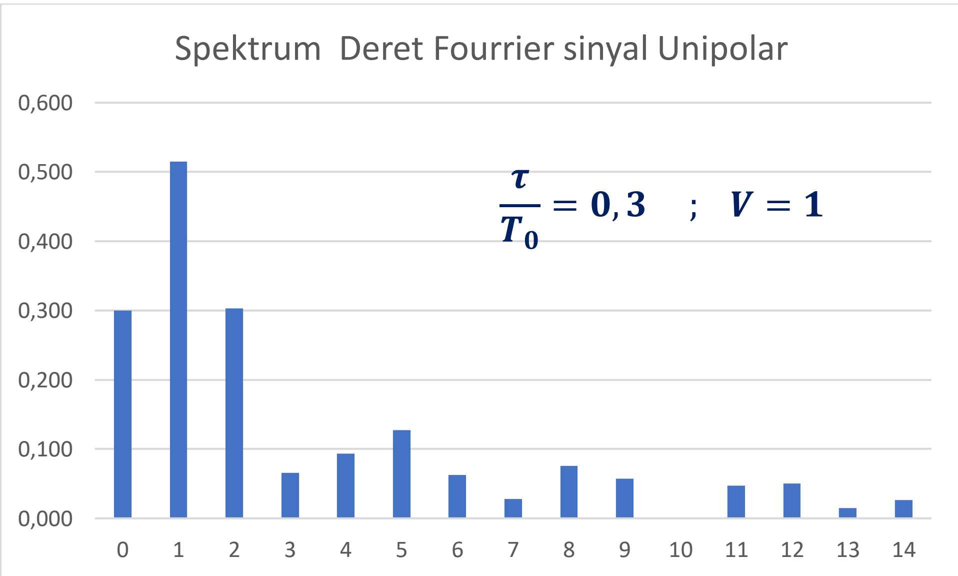
$$x(t) = a_0 + \sum_{n=1}^{\infty} d_n \, \cos(n\omega_0 t - \theta_n) \quad , \quad \theta_n = \tan^{-1}\left(\frac{b_n}{a_n} \right)$$

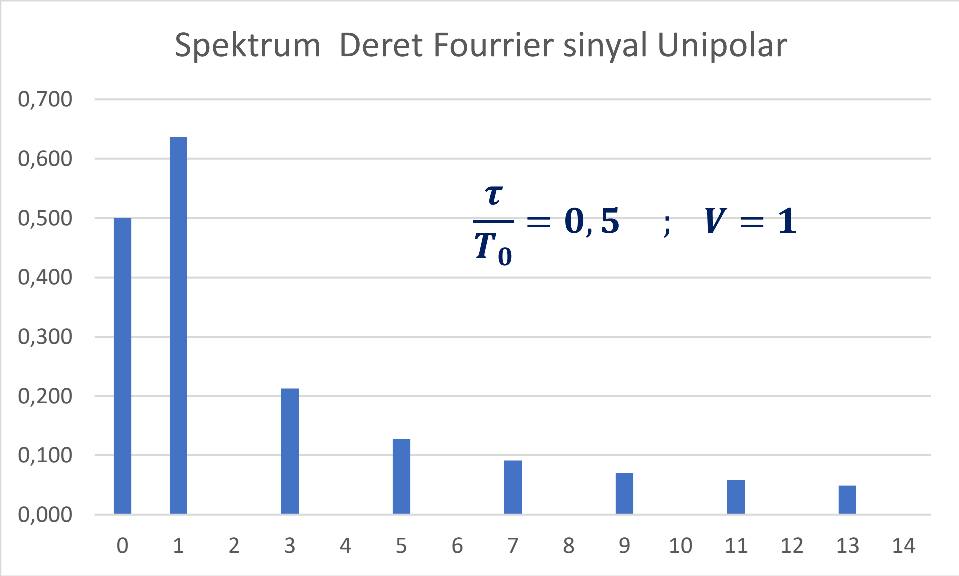
Kurva nilai d_n versus nilai n atau $n\, f_0$ dinamakan **spektrum deret Fourrier**

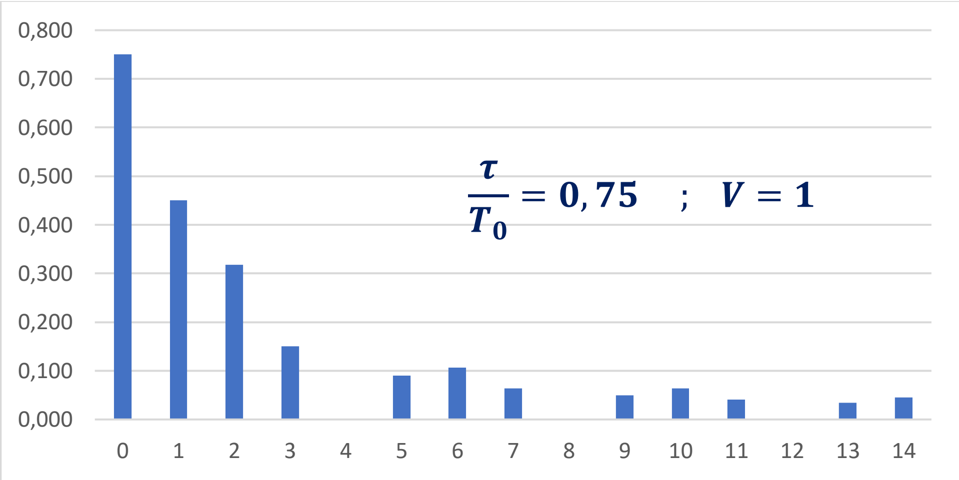
(Kurva spektrum deret fourrier dibawah ini hasil perhitungan menggunakan excel)



$Nilai\ \ a_0\ \ pada\ kurva\ \ ditunjukkan\ \ pada\ \ n = 0$







Sumbu vertical = nilai $\boldsymbol{d_n}$, sumbu horizontal = $\boldsymbol{n f_0} = \frac{\boldsymbol{n}}{\boldsymbol{T_0}}$

$$\boldsymbol{x(t)} = \boldsymbol{a_0} + \sum_{n=1}^{\infty} \boldsymbol{d_n} \cos(n\omega_0 t - \boldsymbol{\theta_n}) \quad , \quad \boldsymbol{\theta_n} = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$$Phasa\ \ sinyal\ harmonik\ (\boldsymbol{\theta_n}) = \begin{cases} \tan^{-1}\left(\frac{b_n}{a_n}\right) & ; \ \boldsymbol{a_n > 0} \\ \tan^{-1}\left(\frac{-b_n}{a_n}\right) & ; \ \boldsymbol{a_n < 0} \end{cases}$$

Sampai pada bahasan ini telah dibuktikan bahwa sinyal segiempat Unipolar pada Gamabr 1 terbentuk dari (terdiri dari) sinyal dc dan sejumlah tak hingga sinyal-sinyal sinusoidal dalam bentuk deret :

$$\boldsymbol{x(t)} = a_0 + \sum_{n=1}^{\infty} \boldsymbol{d_n} \cos(n\omega_0 t - \boldsymbol{\theta_n})$$

Dari Spektrum Deret Fourrier tampak bahwa sinyal x(t) tersebut menduduki Band Width yang sangat lebar .

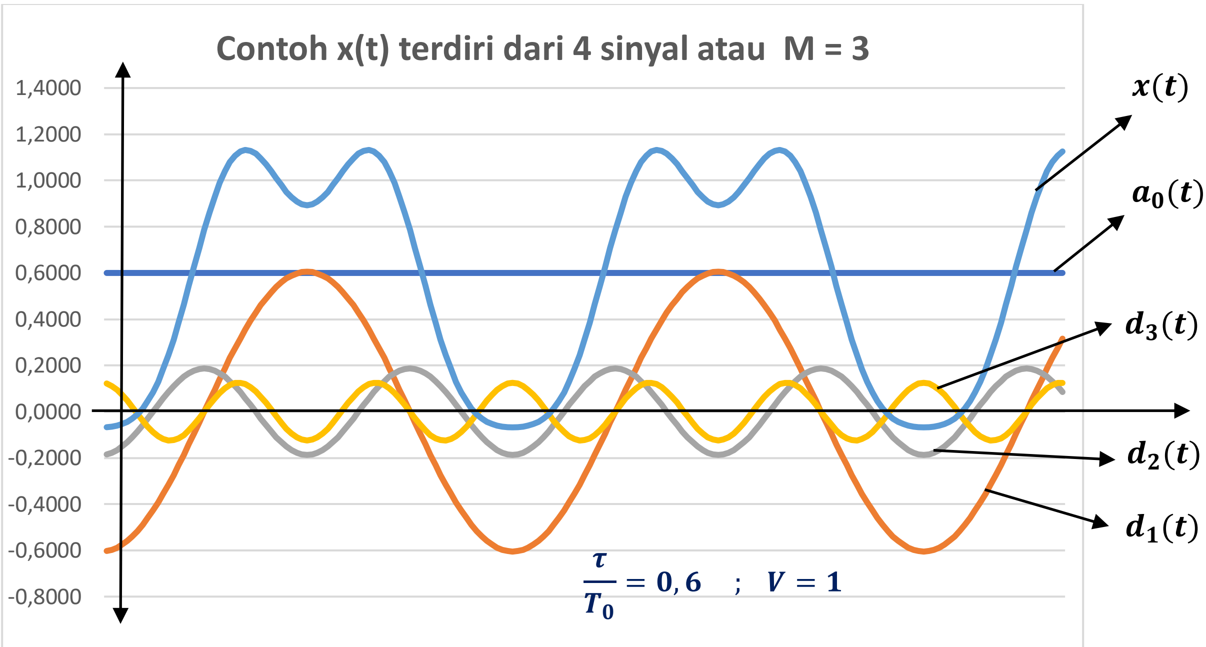
Bagaimana bila jumlah deret dibatasi sbb :

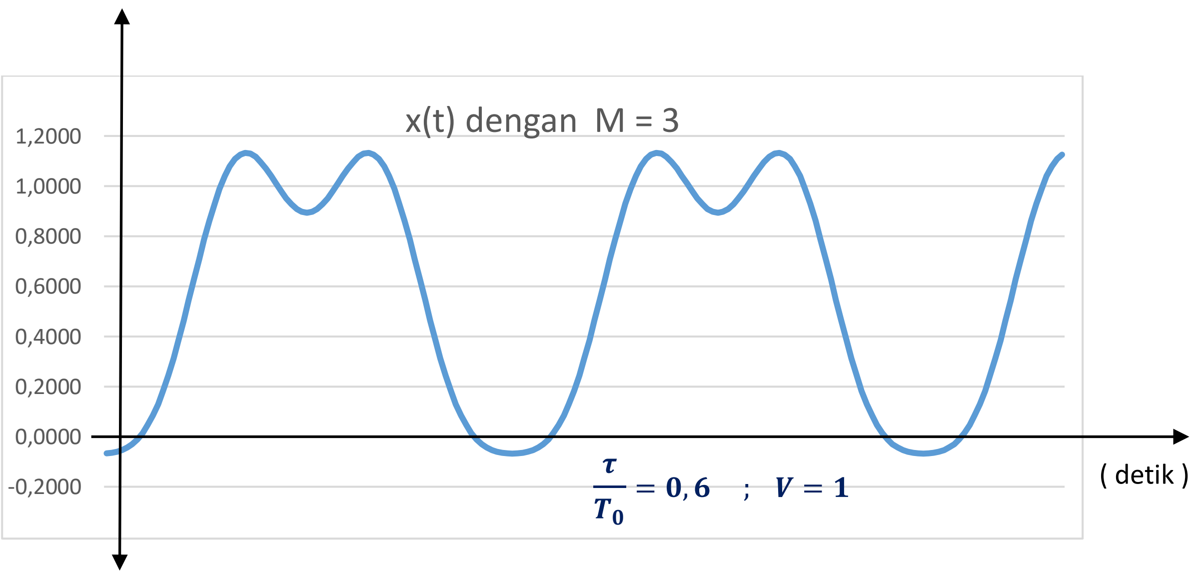
$$\boldsymbol{x(t)} = \boldsymbol{a_0} + \sum_{n=1}^{M} \boldsymbol{d_n} \cos(n\omega_0 t - \boldsymbol{\theta_n}) \quad ; \quad \boldsymbol{M = positif \;\; berhingga}$$

Perhatikan contoh berikut :

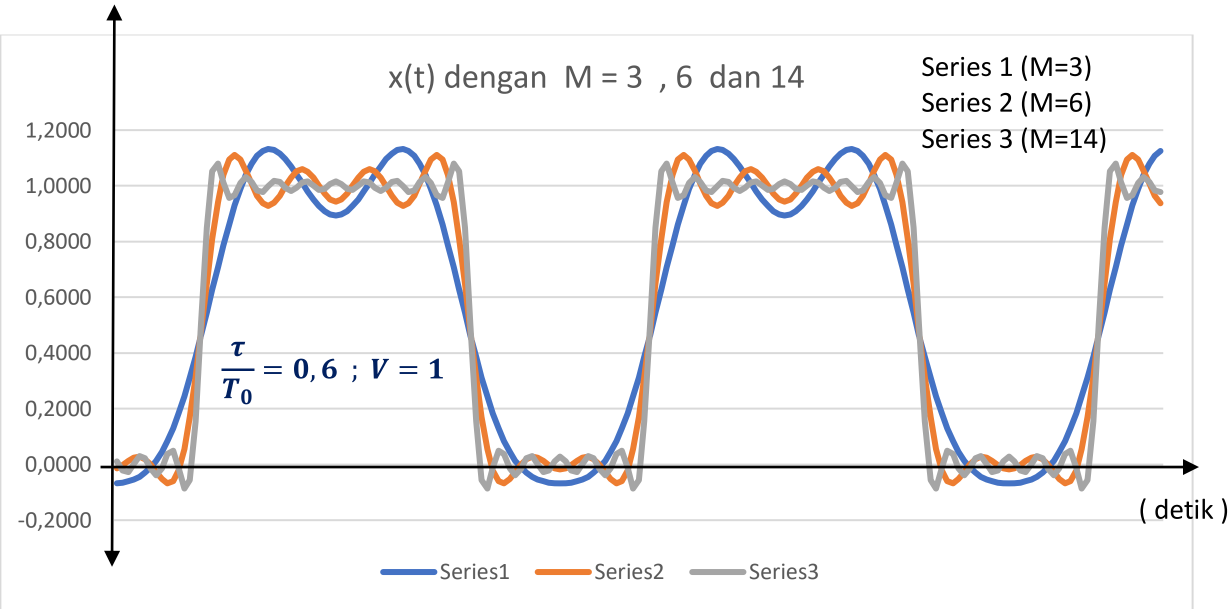
$$a_0(t) = a_0 \quad ; \quad d_1(t) = \boldsymbol{d_1} \cos(\omega_0 t - \boldsymbol{\theta_1}) \quad ; \quad d_2(t) = \boldsymbol{d_2} \cos(2\omega_0 t - \boldsymbol{\theta_2})$$

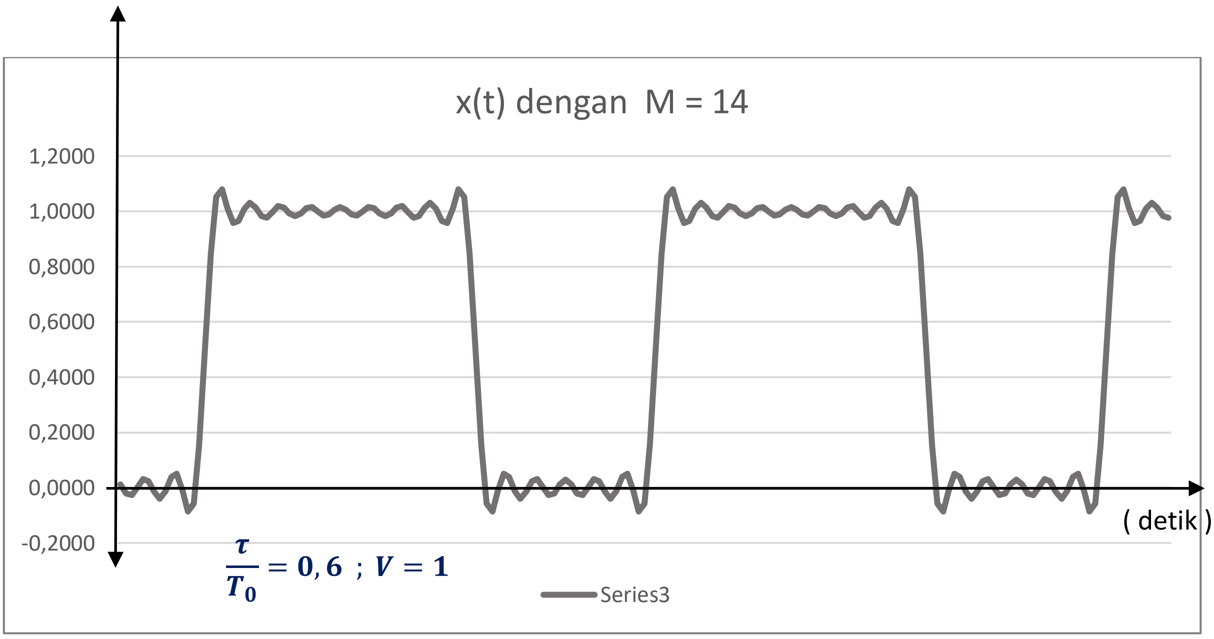
$$d_3(t) = \boldsymbol{d_3} \cos(3\omega_0 t - \boldsymbol{\theta_3}) \quad ; \quad \boldsymbol{x(t)} = \boldsymbol{a_0} + \sum_{n=1}^{M=3} \boldsymbol{d_n} \cos(n\omega_0 t - \boldsymbol{\theta_n})$$



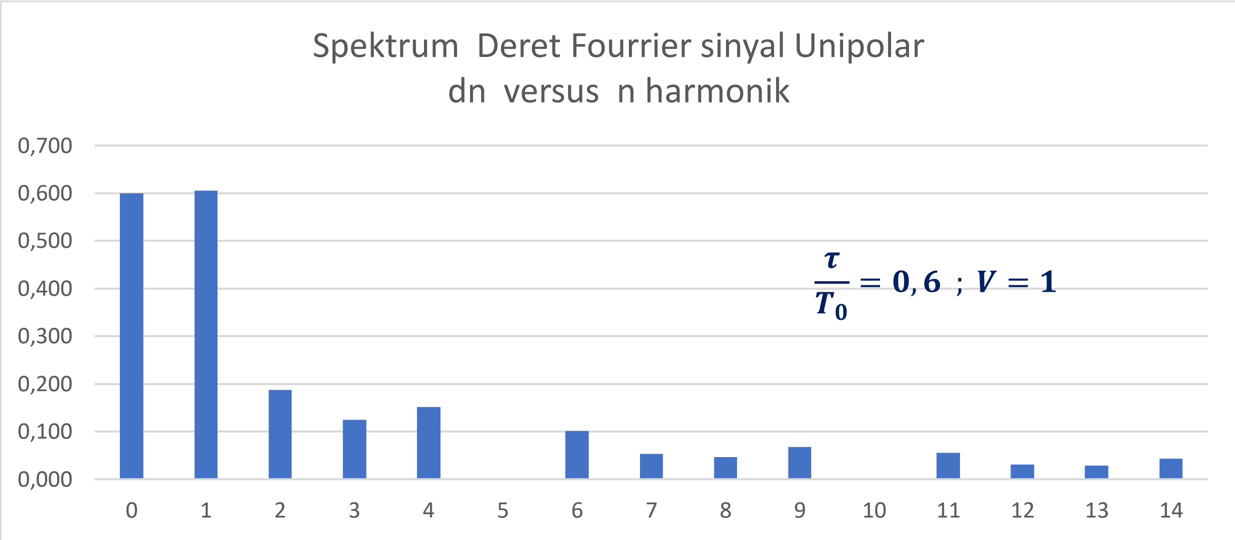


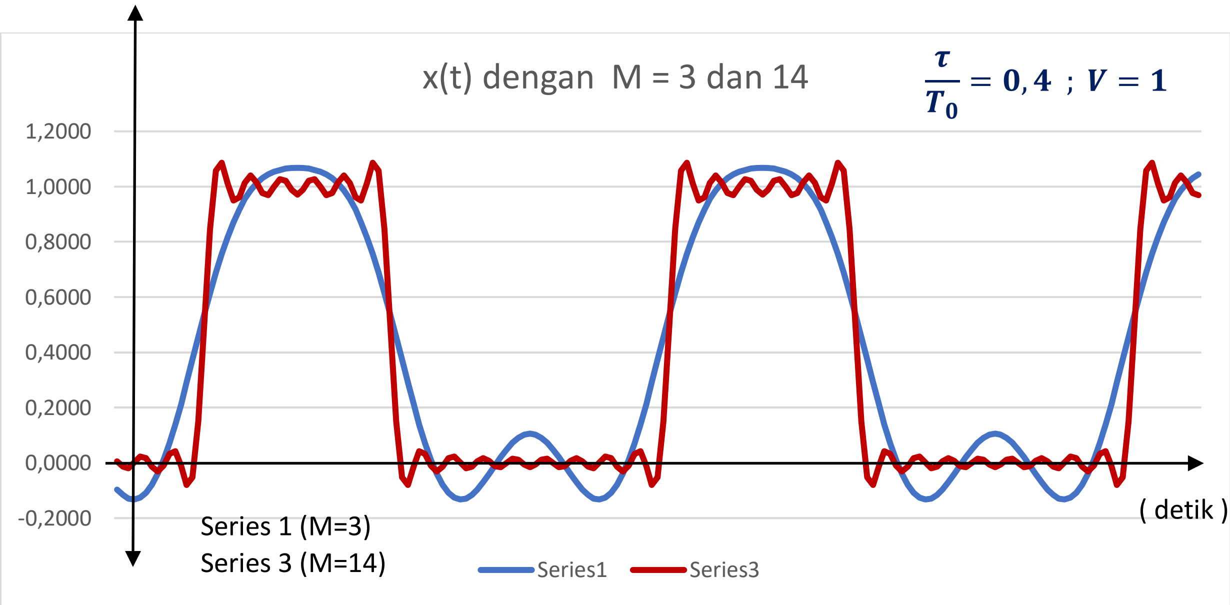
Bila komponen harmonic diperbanyak akan menghasilkan x(t) mendekati bentuk sinyal x(t) dg BW yang sangat besar (tak hingga)

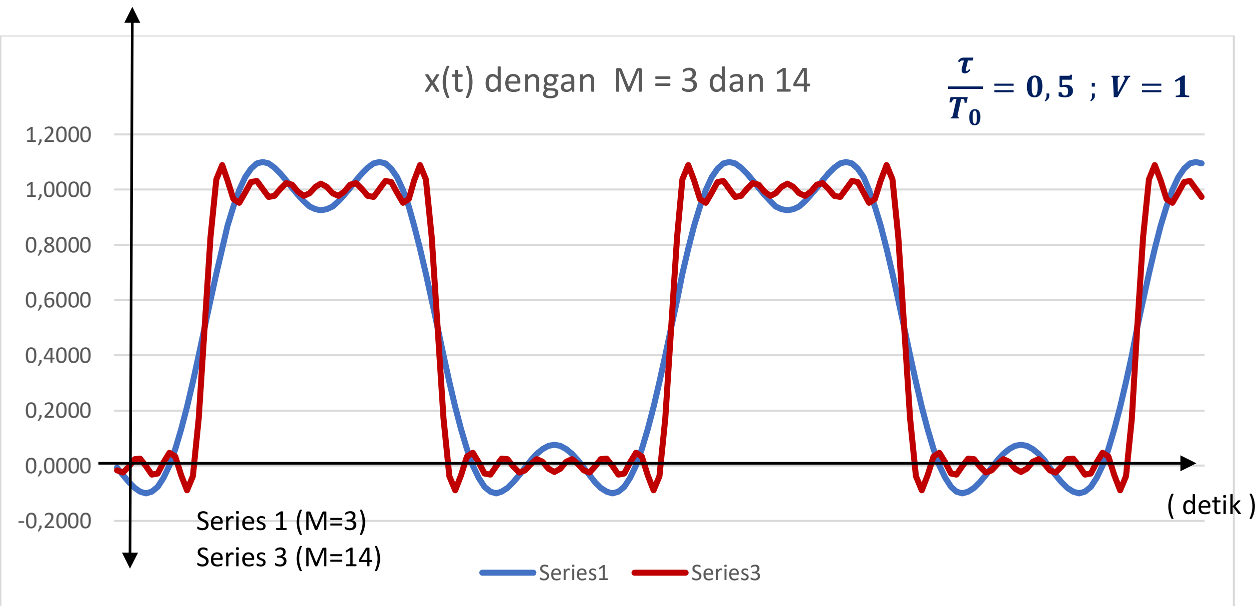


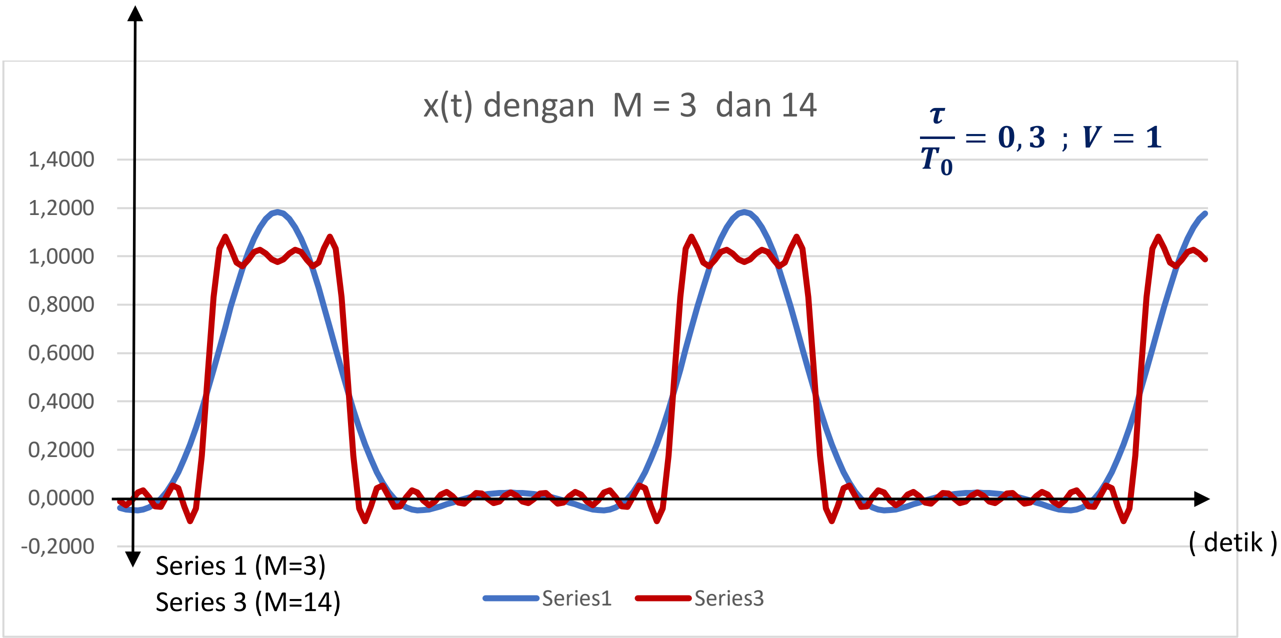


Band Widh yang dibutuhkan = $\boldsymbol{BW = M f_0 = \frac{M}{T_0}}$

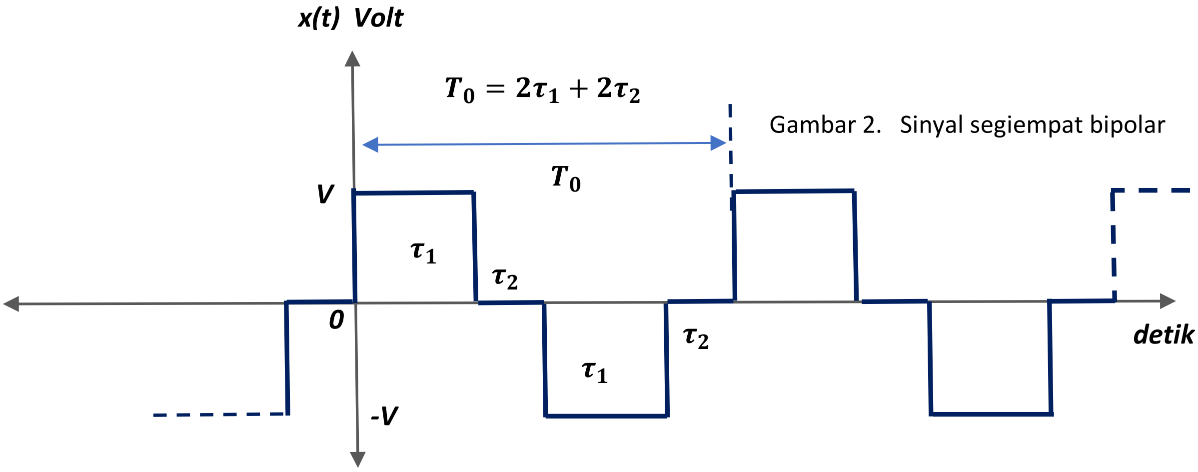

Spektrum Deret Fourrier sinyal Unipolar
dn versus n harmonik
$\frac{\tau}{T_0} = 0,6 \; ; \; V = 1$
0,700
0,600
0,500
0,400
0,300
0,200
0,100
0,000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14


x(t) dengan M = 3 dan 14
$\frac{\tau}{T_0} = 0,4 \; ; \; V = 1$
1,2000
1,0000
0,8000
0,6000
0,4000
0,2000
0,0000
-0,2000
Series 1 (M=3)
Series 3 (M=14)
(detik)
Series1
Series3


x(t) dengan M = 3 dan 14
$\frac{\tau}{T_0} = 0,5 \; ; \; V = 1$
1,2000
1,0000
0,8000
0,6000
0,4000
0,2000
0,0000
-0,2000
Series 1 (M=3)
Series 3 (M=14)
(detik)
Series1
Series3



Contoh 2 : Mendapatkan deret Fourrier sinyal periodic segiempat bipolar



Gambar 2. Sinyal segiempat bipolar

a). Menghitung koefisien Cosinus yaitu : a_0 , a_1 , a_2 , . . . a_n

$$a_0 = \frac{1}{T_0} \int_{t_x}^{t_x+T_0} x(t)\ dt = 0 \quad ; \textit{lihat gambar} \rightarrow T_0 \text{ adalah nilai perioda}$$

$$a_n = \frac{2}{T_0} \int_{t_x}^{t_x+T_0} x(t)\ \cos(n\,\omega_0 t)\ dt \quad ; f_0 = \frac{1}{T_0}\ , \quad \omega_0 = 2\pi\ f_0$$

Nilai t_x dapat dipilih sembarang ,dipilih $t_x = 0$ sehingga : (lihat gambar)

$$a_n = \frac{2}{T_0}\int_0^{\tau_1} V\ \cos(n\omega_0 t)\ dt\ +\ \frac{2}{T_0}\int_{\tau_1+\tau_2}^{\tau_1+\tau_2+\tau_1} (-V)\ \cos(n\omega_0 t)\ dt$$

$$a_n = \frac{2V}{n\,\omega_0\ T_0}\sin(n\omega_0 t)\Big]_0^{\tau_1} + \frac{(-2V)}{n\,\omega_0\ T_0}\sin(n\omega_0 t)\Big|_{\tau_1+\ \tau_2}^{\tau_1+\ \tau_2+\tau_1} \quad ; \quad n > 0$$

$$a_n = \frac{2V}{n\,\omega_0\ T_0}\ \sin(n\omega_0\,\tau_1) + \frac{2V}{n\,\omega_0\ T_0}\ (\sin(n\omega_0\,[\tau_1 +\ \tau_2]\,) - \ \sin(n\omega_0\,[2\tau_1 +\ \tau_2])\,)$$

$$a_n = \frac{2V}{n\,\omega_0\ T_0}\ (\ \sin(n\omega_0\tau_1) +\ \sin(n\omega_0\,[\tau_1 +\ \tau_2]\,) - \ \sin(n\omega_0\,[2\tau_1 +\ \tau_2])\ )$$

$$a_n = \frac{V}{n\,\pi}\left(\ \sin\left(2n\,\pi\ \frac{\tau_1}{T_0}\right) +\ \sin\left(2n\,\pi\ \frac{[\tau_1 +\ \tau_2]}{T_0}\right) - \ \sin\left(2n\,\pi\ \frac{[2\tau_1 +\ \tau_2]}{T_0}\right)\ \right)$$

$$a_n = \frac{V}{n\,\pi}\left(\ \sin\left(2n\,\pi\ \frac{\tau_1}{T_0}\right) - \ \sin\left(2n\,\pi\ \frac{[2\tau_1 +\ \tau_2]}{T_0}\right)\ \right)$$

b). Menghitung koefisien Sinus yaitu : $b_1\ ,\ b_2\ ,\ \ b_3\ ,\ \ .\ .\ .\ b_n$

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\ \sin(n\,\omega_0 t)\ \ dt = \qquad .\ .\ .(\ lihat\ gambar)$$

$$b_n = \frac{2}{T_0}\int_0^{\tau_1} V\ \sin(n\omega_0 t)\ \ dt\ +\ \frac{2}{T_0}\int_{\tau_1+\tau_2}^{\tau_1+\tau_2+\tau_1} (-V)\ \sin(n\omega_0 t)\ \ dt$$

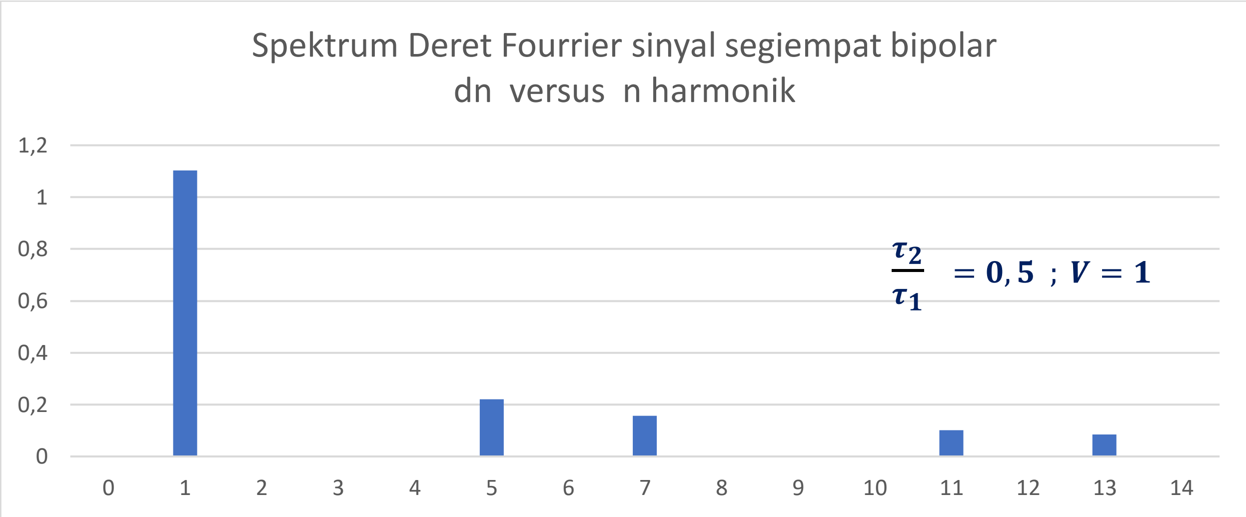
$$b_n = -\frac{2V}{n\omega_0\ T_0}\ \ \cos(\omega_0 t)\Big]_0^{\tau_1} +\ \ \frac{2V}{n\omega_0\ T_0}\ \ \cos(\omega_0 t)\Big|_{\tau_1+\ \tau_2}^{\tau_1+\ \tau_2+\tau_1}$$

$$b_n = \frac{2V}{n\,\omega_0\ T_0}\ (\ [1 - \ \cos(n\omega_0\,\tau_1)\,]$$
$$+\ \ [\cos(n\omega_0\,[2\tau_1 +\ \tau_2]\,)\ - \ \cos(n\omega_0\,[\tau_1 +\ \tau_2])\,]\ \ )$$

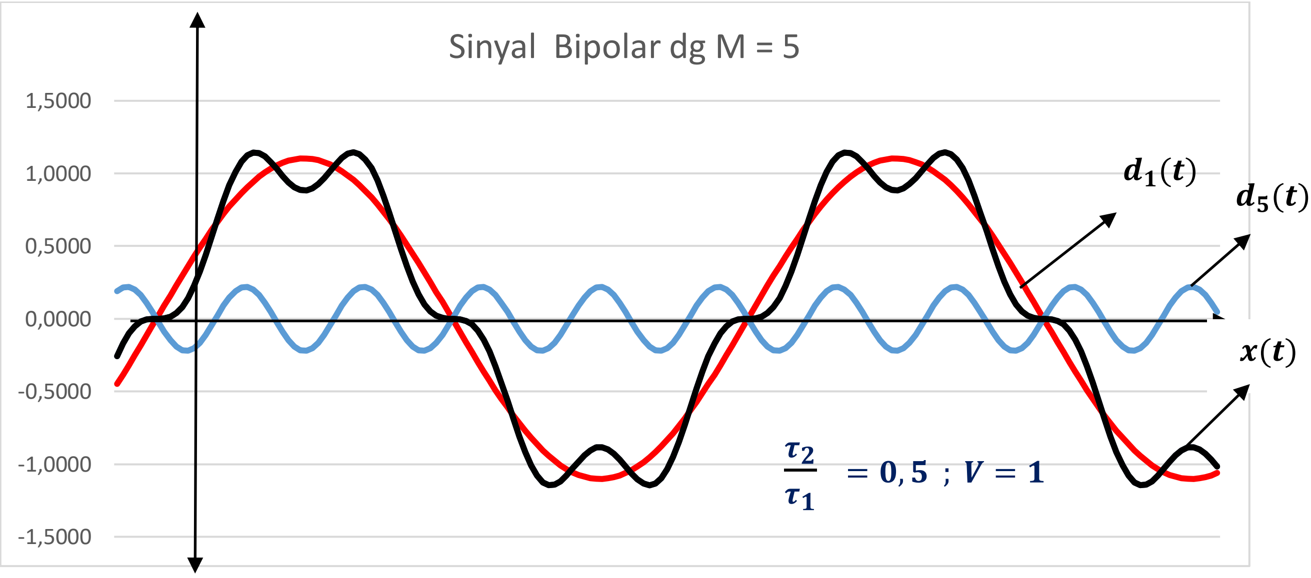
$$b_n = \frac{2V}{n\,\omega_0\ T_0}\ (1 - \cos(n\omega_0\,\tau_1)\ \ +\ \cos(n\omega_0\,[2\tau_1 +\ \tau_2]\,)\ \ - \ \cos(n\omega_0\,[\tau_1 +\ \tau_2])\ )$$

$$b_n = \frac{V}{n\,\pi}\left(1 + \cos\left(2n\,\pi\ \frac{[2\tau_1 +\ \tau_2]}{T_0}\right) - \ \cos\left(2n\,\pi\ \frac{[\tau_1 +\ \tau_2]}{T_0}\right)\ - \ \cos\left(2n\,\pi\ \frac{\tau_1}{T_0}\right)\ \right)$$

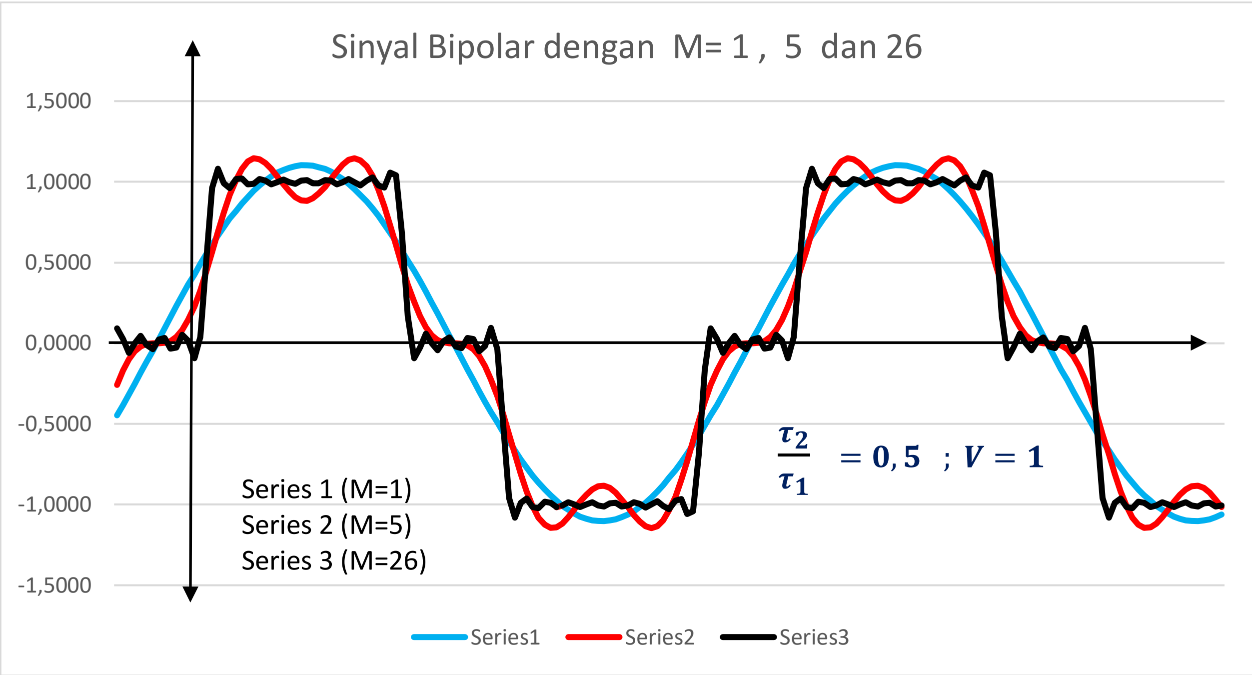
$$b_n = \frac{V}{n\,\pi}\left(1 - \cos(n\,\pi\ ) + \ \cos\left(2n\,\pi\ \frac{[2\tau_1 + \ \tau_2]}{T_0}\right) - \ \cos\left(2n\,\pi\ \frac{\tau_1}{T_0}\right)\ \right)$$

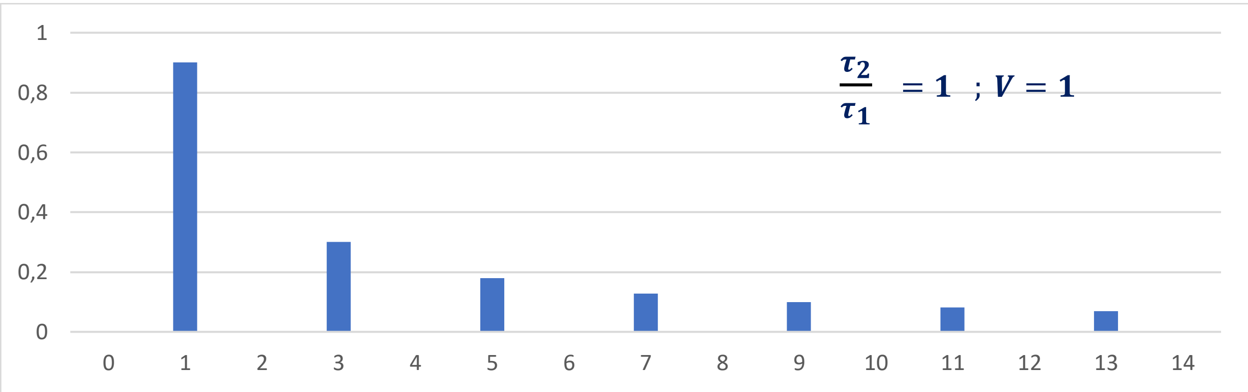


Dari kurva spektrum : $a_0(t) = \ d_2(t) = \ d_3(t) = d_4(t) = \ d_6(t) = \ d_8(t) = \ d_9(t) = \ \ 0$

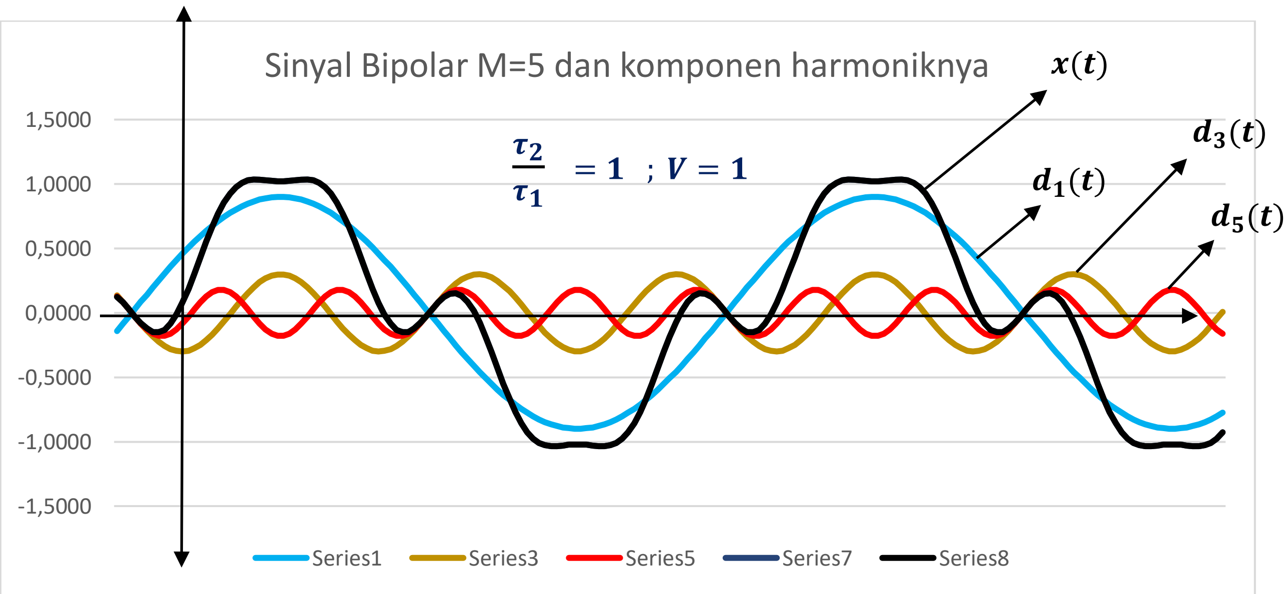


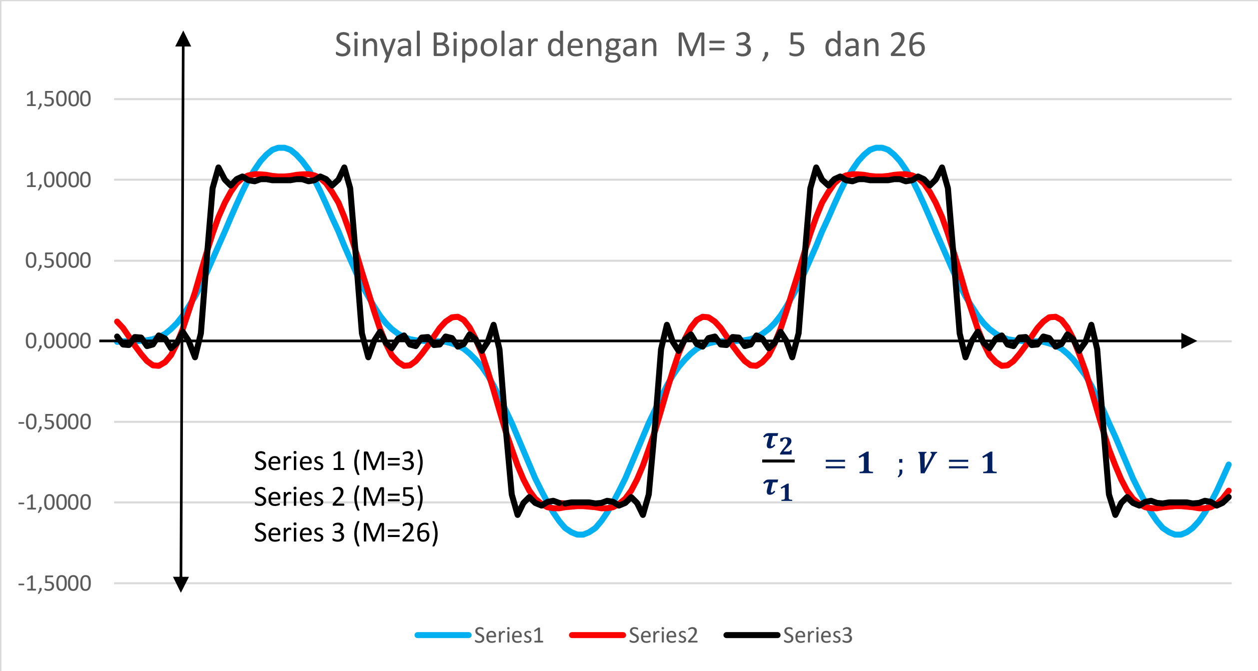
Perhatikan bahwa sinyal ***x(t) untuk M = 5*** dengan $\frac{\tau_2}{\tau_1} = 0,5$ adalah dibentuk oleh **2 buah** sinyal sinussoidal

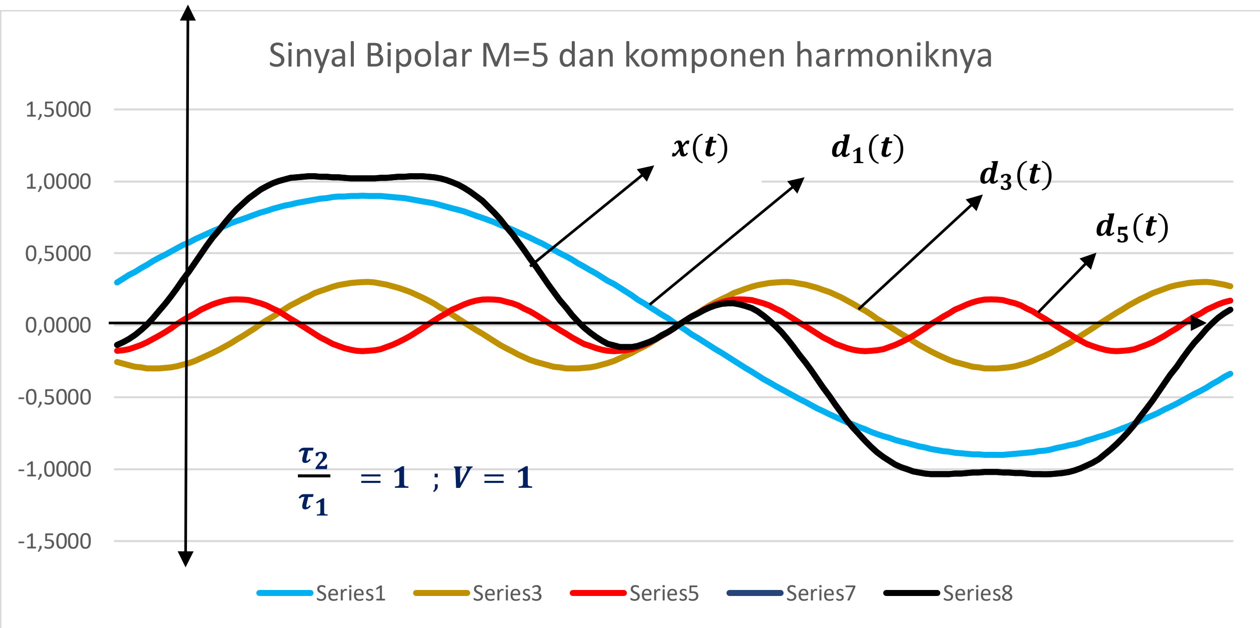




Sumbu vertical = nilai d_n , sumbu horizontal = $n\, f_0 = \frac{n}{T_0}$







Perhatikan bahwa sinyal ***x(t)* *untuk M = 5*** dengan $\frac{\tau_2}{\tau_1} = 1$ adalah dibentuk oleh **3 buah** sinyal sinusoidal

Tabel berikut adalah contoh hasil perhitungan untuk nilai $\frac{\tau_2}{\tau_1} = 1 \;\; dan \;\; V = 1 \; Volt$:

n	Nilai a_n	Nilai b_n	Nilai d_n	Fasa d_n (radian)	Fasa d_n (derajat)
0	0,0000	0,0000	0	0	0
1	0,6366	0,6366	0,9003163	0,7854	45,0000
2	0,0000	0,0000	0	0,0000	0,0000
3	-0,2122	0,2122	0,3001054	0,7854	45,0000
4	0,0000	0,0000	0	0,0000	0,0000
5	0,1273	0,1273	0,1800633	0,7854	45,0000
6	0,0000	0,0000	0	0,0000	0,0000
7	-0,0909	0,0909	0,1286166	0,7854	45,0000
8	0,0000	0,0000	0	0,0000	0,0000
9	0,0707	0,0707	0,1000351	0,7854	45,0000
10	0,0000	0,0000	0	0,0000	0,0000
11	-0,0579	0,0579	0,0818469	0,7854	45,0000
12	0,0000	0,0000	0	0,0000	0,0000
13	0,0490	0,0490	0,0692551	0,7854	45,0000
14	0,0000	0,0000	0	0,0000	0,0000

$$Untuk \;\; M = 5 \;\; maka \;\; : \;\; x(t) \; = \; a_0 + \sum_{n=1}^{M=5} d_n \;\; \cos(n\omega_0 t - \; \theta_n)$$

Atau :

$$x(t) = \;\; a_0 + \sum_{n=1}^{M=5} a_n \cos(n\omega_0 t\,) \; + \; b_n \sin\,(n\omega_0 t\,)$$

2. DERET FOURRIER EKSPONENSIAL

Dari rumus identitas Euleur : $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

Maka Deret Fourrier dapat dinyatakan sbb :

$$x(t) = \;\; a_0 + \sum_{n=1}^{\infty} d_n \cos(n\omega_0 t - \; \theta_n)$$

$$= \;\; a_0 + \sum_{n=1}^{\infty} d_n \left[\frac{e^{j(n\omega_0 t - \,\theta_n)} + \; e^{-j(n\omega_0 t - \,\theta_n)}}{2}\right]$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{d_n}{2} \; e^{j\theta_n} \right) e^{j(n\omega_0 t)} \quad ; \quad (\; Buktikan \;\; sebagai \;\; latihan \;)$$

$$x(t) \;=\; \sum_{n=-\infty}^{\infty} c_n \, e^{j(n\omega_0 t)} \quad ; \quad c_n = \frac{\sqrt{(a_n)^2 + (b_n)^2}}{2} \;\; e^{j\theta_n}$$

$$c_n = \frac{\sqrt{(a_n)^2 + (b_n)^2}}{2} \;\; e^{j\theta_n} \;=\; \frac{a_n - jb_n}{2} \quad ; \quad |c_n| = \frac{\sqrt{(a_n)^2 + (b_n)^2}}{2}$$

$$c_n = \frac{1}{T_0} \int_{t_x}^{t_x+T_0} x(t) \;\, e^{-j(n\omega_0 t)} \; dt \quad ; \quad (\; Buktikan \;\; sebagai \;\; latihan - buka \; text \; Book \;)$$

d_n = koefisien Deret Fourrier sinusoidal = $\sqrt{(a_n)^2 + (b_n)^2}$

$|c_n|$ = koefisien Deret Fourrier eksponensial = $\frac{d_n}{2}$

$$\textbf{Deret Fourrier Eksponensial} \;\; \rightarrow x(t) \;=\; \sum_{n=-\infty}^{\infty} c_n \, e^{j(n\omega_0 t)}$$

3. TRANSFORMASI FOURRIER

Dari Deret Fourrier Eksponensial :

$$x(t) \;=\; \sum_{n=-\infty}^{\infty} c_n \, e^{j(n\omega_0 t)} \;=\; \sum_{n=-\infty}^{\infty} \left[\frac{1}{T_0} \int_{t_x}^{t_x+T_0} x(t) \;\, e^{-j(n\omega_0 t)} \; dt \right] e^{j(n\omega_0 t)}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{T_0} \int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} x(t) \;\, e^{-j(n\omega_0 t)} \; dt \right] e^{j(n\omega_0 t)} =$$

$$= \sum_{n=-\infty}^{\infty} \left[f_0 \int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} x(t) \;\, e^{-j(n\omega_0 t)} \; dt \right] e^{j(n\omega_0 t)}$$

$$= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \left[2\pi f_0 \int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} x(t)\; e^{-j(n\omega_0 t)}\; dt \right] e^{j(n\omega_0 t)}$$

$$= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \left[\omega_0 \int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} x(t)\; e^{-j(n\omega_0 t)}\; dt \right] e^{j(n\omega_0 t)}$$

$$Bila\;\; T_0 \to +\infty \;\; maka : \;\; X(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \left[\Delta\omega \int_{-\infty}^{+\infty} x(t)\; e^{-j(n\Delta\omega\, t)}\; dt \right] e^{j(n\Delta\omega\, t)}$$

$$Dari\;\; Kalkulus\;\; maka : \;\; X(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[d\omega \int_{-\infty}^{+\infty} x(t)\; e^{-j(\omega\, t)}\; dt \right] e^{j(\omega\, t)}$$

$$Dapat\; dituliskan \;\; : \;\; X(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} x(t)\; e^{-j(\omega t)}\; dt \right] e^{j(\omega\, t)}\;\; d\omega$$

$$Dapat\; juga\;\; dituliskan \;\; : \;\; X(t) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} x(t)\; e^{-j(2\pi f\, t)}\; dt \right] e^{j(2\pi f\, t)}\;\; df$$

$$\int_{-\infty}^{+\infty} x(t)\; e^{-j(2\pi f\, t)}\; dt \;=\; hasilnya\;\; fungsi\;\; (f) \to X(f)$$

$$X(f)\;\; inilah\;\; yang\; disebut\;\; Trans\; Fourrier\;\; dari\;\; x(t)$$

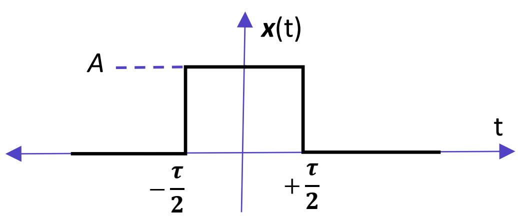
$$Perhatikan \;\; : \;\; x(t) = \int_{-\infty}^{+\infty} [\, X(f)\,] e^{j(2\pi f\, t)}\;\; df \;\to\;\; disebut\;\; invers\;\; Trans\;\; Fourrier$$

$$Trans\; Fourrier\; dari\;\; s(t) \;\; : \;\; TF[\, s(t)\,] = S(f) = \int_{-\infty}^{+\infty} s(t)\;\; e^{-j(2\pi f\, t)}\;\; dt$$

$$Invers\;\; TF\; dari\;\; S(f)\;\; adalah : \;\;\; s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{j(2\pi f\, t)}\;\; df$$

Beberapa contoh menghitung Trans Fourrier .

1). Sinyal pulsa segi empat



Gambar. 1A

$$x(t) = \begin{cases} A \;\; ; & -\frac{\tau}{2} \le t \le +\frac{\tau}{2} \\ 0 \;\; ; & |t| > \frac{\tau}{2} \end{cases}$$

sering ditulikan :

$$x(t) = \;\; A\, rect\left(\frac{t}{\tau}\right)$$

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f\, t}\ dt = \int_{-0.5\,\tau}^{+0.5\,\tau} A\, e^{-j2\pi f\, t}\ dt$$

$$X(f) = \frac{A}{-\,j2\pi f} \int_{-0.5\,\tau}^{+0.5\,\tau} e^{-j2\pi f\, t}\ d(\,-j2\pi f\ t\,)\ ;$$

$$\boxed{ingat\ \rightarrow \int e^x\, dx = e^x + C}$$

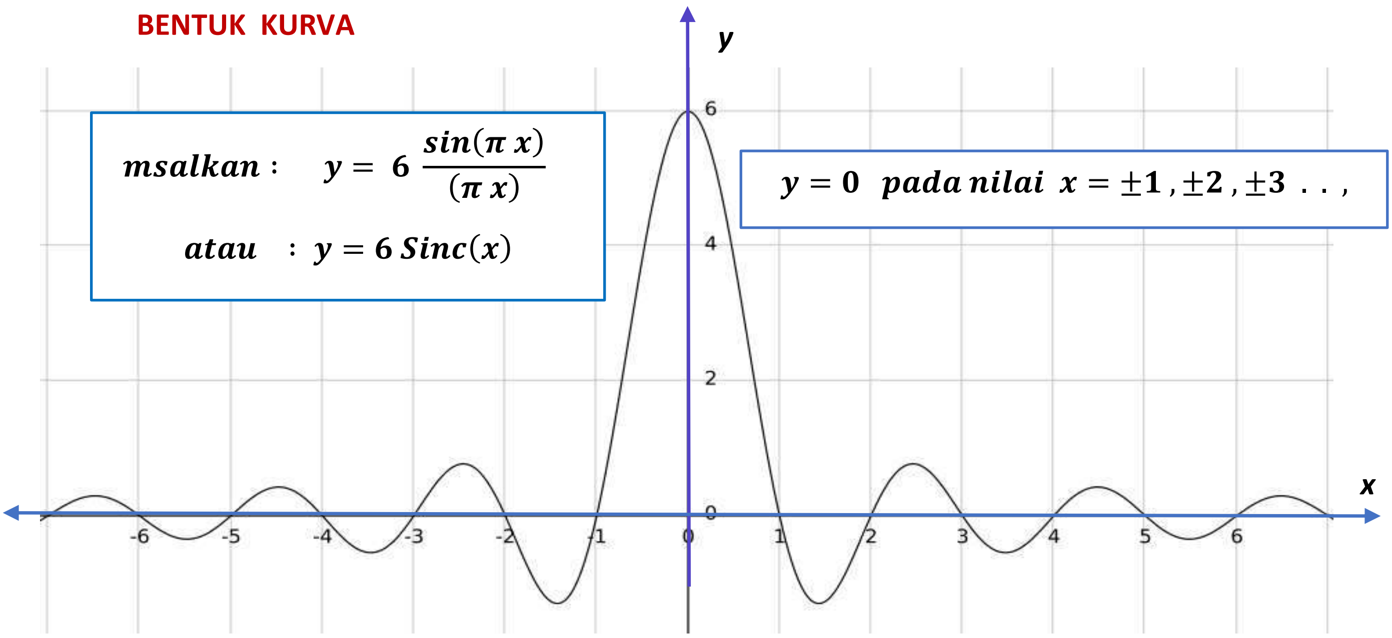
$$X(f) = \frac{A}{-\,j2\pi f}\left(e^{-j2\pi f\,[+0.5\,\tau]} - e^{-j2\pi f\,[-0.5\,\tau]}\right) = X(f) = \frac{A}{-\,j2\pi f}\left(e^{-j\pi f\tau} - e^{j2\pi f\tau}\right) =$$

$$X(f) = \frac{A}{j2\pi f}\left(e^{j\pi f\tau} - e^{-j\pi f\tau}\right) \quad ; \quad Euleur \rightarrow \begin{cases} e^{jx} = \cos x + j\sin x \\ e^{-jx} = \cos x - j\sin x \\ \boldsymbol{\cos x = \dfrac{(e^{jx} + e^{-jx})}{2}} \\ \boldsymbol{\sin x = \dfrac{(e^{jx} - e^{-jx})}{2j}} \end{cases}$$

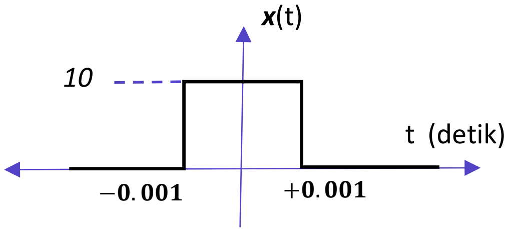
$$maka\ :\ \boldsymbol{X(f) = \frac{A\,\sin(\pi f\tau)}{\pi f}} \quad ; \quad Definisi\ fungsi\ \boldsymbol{Sinc(x)} \rightarrow \boldsymbol{Sinc(x)} = \frac{\sin(\,\pi\, x\,)}{\pi\, x}$$

$$\boldsymbol{TF}\left[\boldsymbol{A\, rect}\left(\frac{t}{\tau}\right)\right] = A\tau\,\frac{\sin(\pi\ f\tau)}{\pi\ f\tau} = A\tau\,\boldsymbol{Sinc}(f\tau) \quad .\ .\ .\ .\ .\ (1)$$

BENTUK KURVA

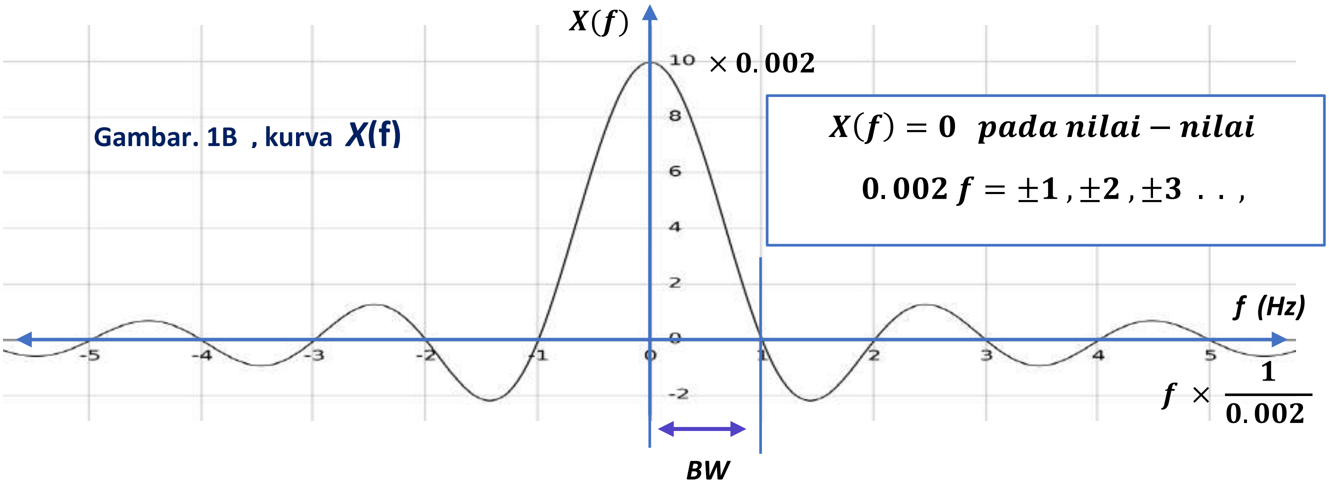


Misal nilai A = 10 Volt , $\tau = 0.002$ detik , atau dpt dituliskan : $x(t) = 10\,\boldsymbol{rect}\left(\frac{t}{0.002}\right)$



$$X(f) = 10 \times 0.002\,\frac{\sin(0.002\,\pi\, f\,)}{0.002\,\pi\, f}$$

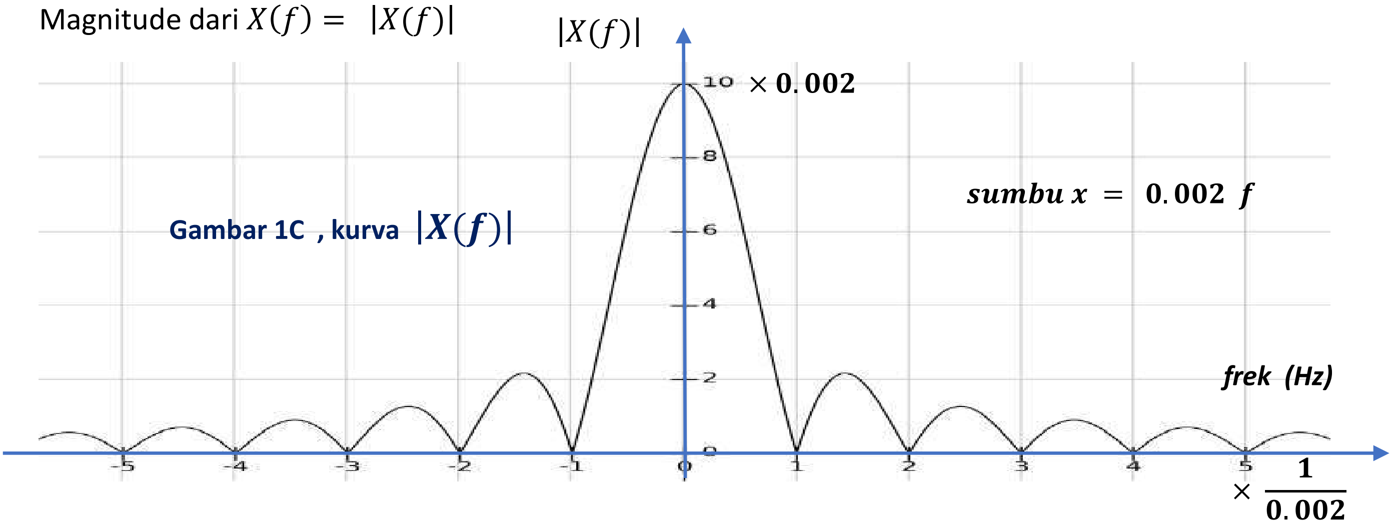
$$= 0.02\,\frac{\sin(0.002\,\pi\, f\,)}{0.002\,\pi\, f}$$



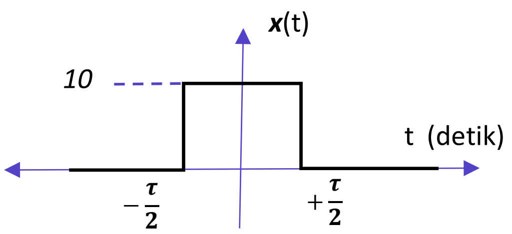
BW satu sisi (frek positif saja) pada kasus gambar di atas adalah ***Null BW satu sisi***

$$BW = \frac{1}{0.002} = \ \mathbf{500\ \ Hz}$$

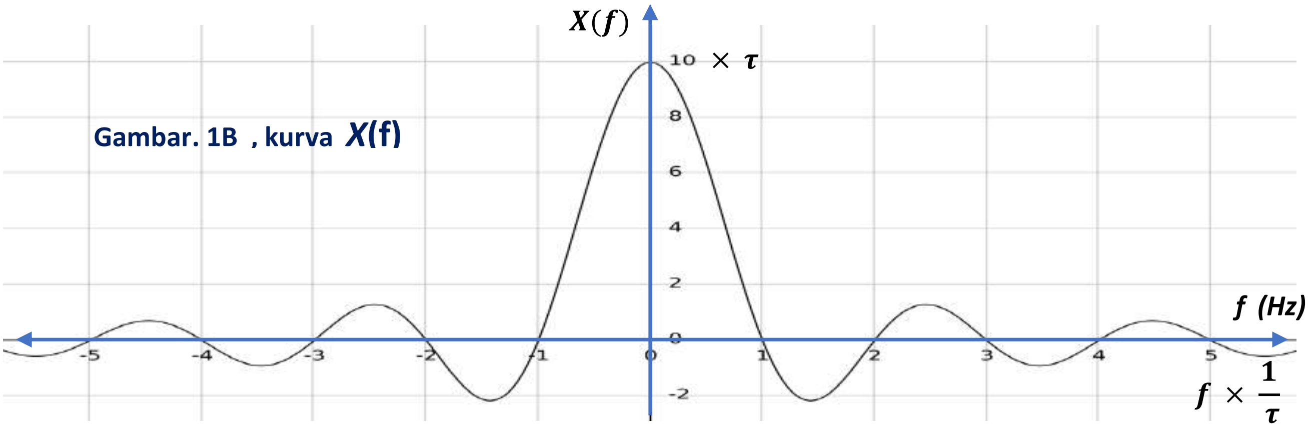
Magnitude dari $X(f) = \ |X(f)|$



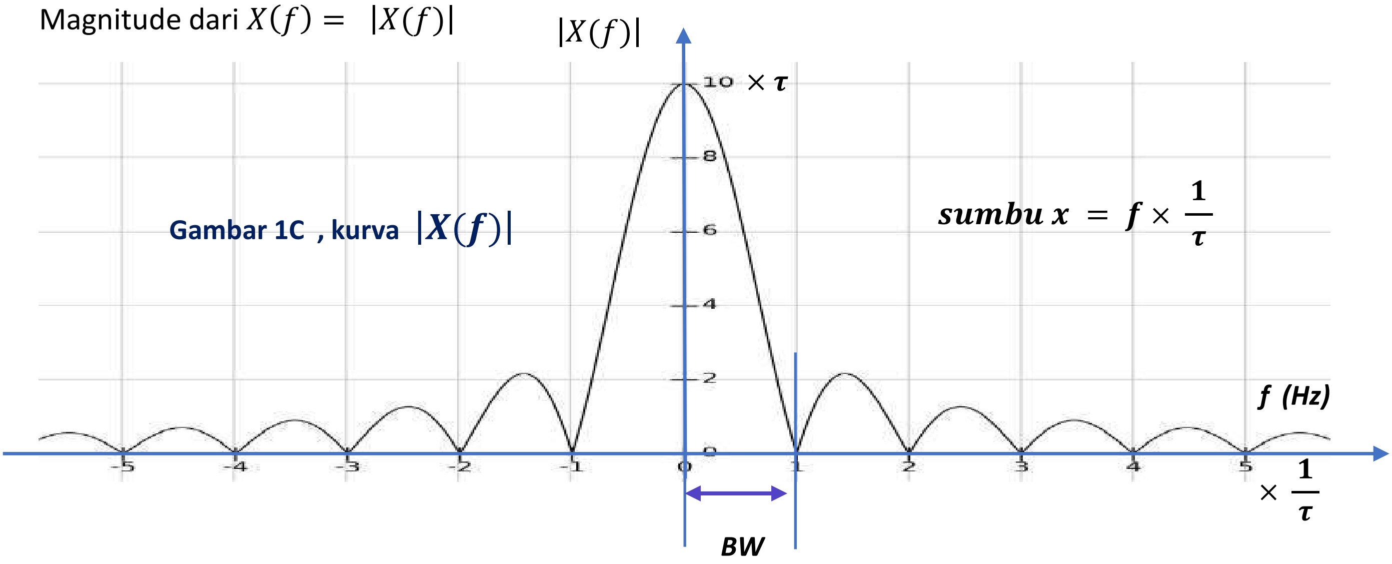
Jadi bila : $x(t) = 10\, \boldsymbol{rect}\left(\frac{t}{\tau}\right)$



$$X(f) = \ 10\, \tau\ \frac{\sin(\pi\, f\, \tau)}{\pi\, f\ \ \tau}$$



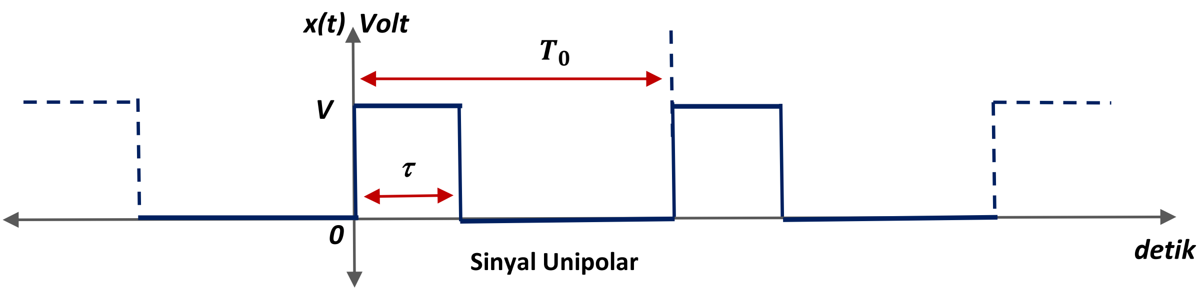
Magnitude dari $X(f) = |X(f)|$

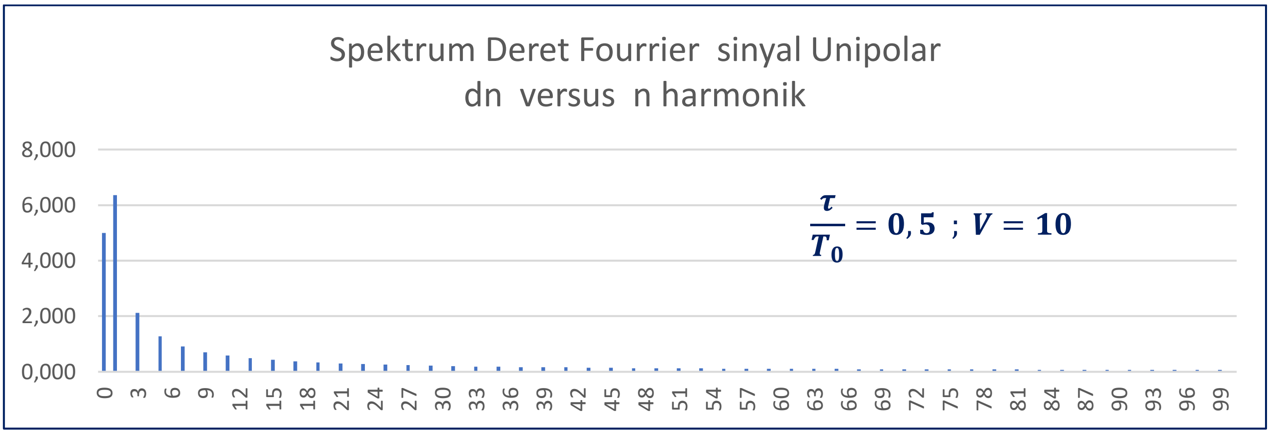


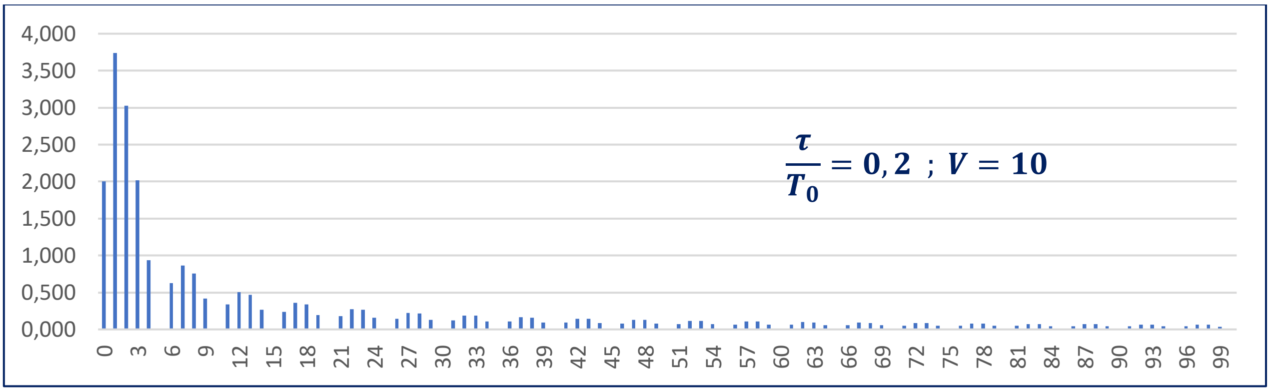
$$\textit{Null BW satu sisi} = \frac{1}{\tau} \text{ Hz}$$

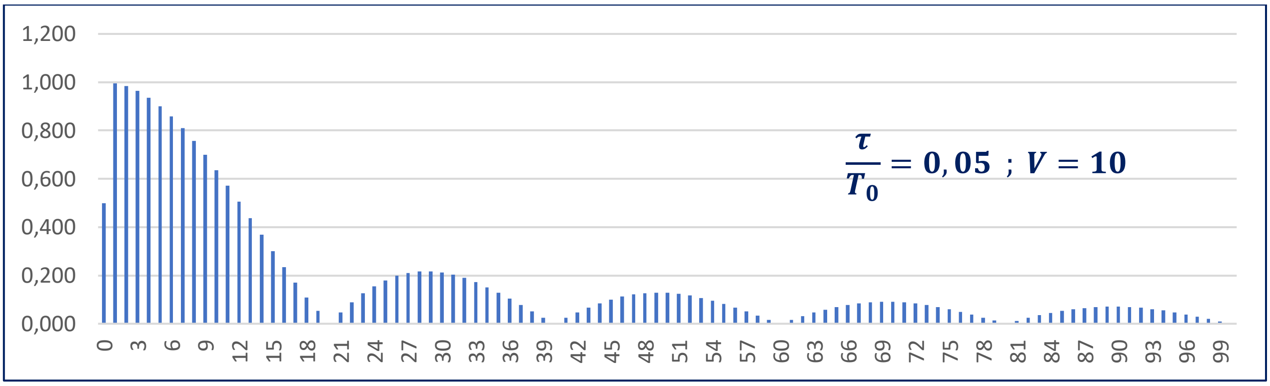
Perhatikan bahwa besar Null Band Width hanya dipengaruhi oleh lebat pulsa (τ)

Bandingkan dengan spektrum Deret Fourrier sinyal periodic Unipolar berikut ini









Sumbu vertical = nilai d_n , sumbu horizontal = $n f_0 = \frac{n}{T_0}$

$$d_n = \frac{V}{\pi\, n} \sqrt{2 - 2 \cos\left(n\, 2\pi\, \frac{\tau}{T_0}\right)} \quad ; \quad a_0 = \frac{V}{T_0}\, \tau$$

Perhatikan bahwa bila nilai T_0 semakin besar akan menghasilkan spektrum makin rapat

Bandingkan antara spektrum Deret Fourrier dengan Spektrum Transformasi Fourrier

Tabel-1. Pasangan transformasi Fourier.

Sinyal	$f(t)$	$F(\omega)$
Impuls	$\delta(t)$	1
Sinyal searah (konstan)	1	$2\pi\,\delta(\omega)$
Fungsi anak tangga	$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
Signum	$sgn(t)$	$\frac{2}{j\omega}$
Exponensial (kausal)	$\left(e^{-\alpha t}\right)u(t)$	$\frac{1}{\alpha + j\omega}$
Eksponensial (dua sisi)	$e^{-\alpha \lvert t \rvert}$	$\frac{2\alpha}{\alpha^2 + \omega^2}$
Eksponensial kompleks	$e^{j\beta t}$	$2\pi\,\delta(\omega - \beta)$
Cosinus	$\cos\beta t$	$\pi\left[\delta(\omega-\beta) + \delta(\omega+\beta)\right]$
Sinus	$\sin\beta t$	$-j\pi\left[\delta(\omega-\beta) - \delta(\omega+\beta)\right]$

Tabel-2. Sifat-sifat transformasi Fourier.

Sifat	Kawasan Waktu	Kawasan Frekuensi
Sinyal	$f(t)$	$F(\omega)$
Kelinieran	$A\,f_1(t) + B\,f_2(t)$	$AF_1(\omega) + BF_2(\omega)$
Diferensiasi	$\frac{df(t)}{dt}$	$j\omega F(\omega)$
Integrasi	$\int_{-\infty}^{t} f(x)dx$	$\frac{F(\omega)}{j\omega} + \pi\,F(0)\,\delta(\omega)$
Kebalikan	$f(-t)$	$F(-\omega)$
Simetri	$F(t)$	$2\pi\,f(-\omega)$
Pergeseran waktu	$f(t - T)$	$e^{-j\omega T}F(\omega)$
Pergeseran frekuensi	$e^{j\beta t} f(t)$	$F(\omega - \beta)$
Penskalaan	$\lvert a \rvert\, f(at)$	$F\left(\frac{\omega}{a}\right)$

Contoh berkaitan dengan :

1). Materi pergeseran waktu

2). Materi pergeseran frekuensi