CHAPTER 4
HYPOTHESIS TESTING
(MEAN, PROPORTION & VARIANCE)

Hypothesis Testing for the Mean

Hypothesis testing for the mean aims to determine whether there is a difference in the mean of a population with a certain mean or whether there is a difference between the means of one population and another population. A summary of hypothesis testing for the mean is as follows:

<table>
<tr><th>H_0</th><th>Statistical Test</th><th>H_1</th><th>Critical region</th></tr>
<tr><td colspan="4">One Population</td></tr>
<tr><td rowspan="2">$\mu = \mu_0$</td><td>$Z = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$
σ known and/or $n \geq 30$</td><td>$\mu \neq \mu_0$
$\mu > \mu_0$
$\mu < \mu_0$</td><td>$Z < -z_{\alpha/2}$ and $Z > z_{\alpha/2}$
$Z > z_\alpha$
$Z < -z_\alpha$</td></tr>
<tr><td>$T = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$
σ unknown and $n < 30$</td><td>$\mu \neq \mu_0$
$\mu > \mu_0$
$\mu < \mu_0$</td><td>$T < -t_{\alpha/2}$ and $T > t_{\alpha/2}$
$T > t_\alpha$
$T < -t_\alpha$</td></tr>
<tr><td colspan="4">Two Populations</td></tr>
<tr><td>$\mu_1 - \mu_2 = d_0$</td><td>$Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
σ known and/or $n \geq 30$</td><td>$\mu_1 - \mu_2 \neq d_0$
$\mu_1 - \mu_2 > d_0$
$\mu_1 - \mu_2 < d_0$</td><td>$Z < -z_{\alpha/2}$ and $Z > z_{\alpha/2}$
$Z > z_\alpha$
$Z < -z_\alpha$</td></tr>
<tr><td>$\mu_1 - \mu_2 = d_0$</td><td>1. σ unknown and approximated $\sigma_1^2 = \sigma_2^2$
$T = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$
$S_p = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$
$v = n_1 + n_2 - 2$
2. σ unknown and approximated $\sigma_1^2 \neq \sigma_2^2$
$T = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$</td><td>$\mu_1 - \mu_2 \neq d_0$
$\mu_1 - \mu_2 > d_0$
$\mu_1 - \mu_2 < d_0$</td><td>$T < -t_{\alpha/2}$ and T> $t_{\alpha/2}$
$T > t_\alpha$
$T < -t_\alpha$</td></tr>
</table>

	$v = \dfrac{\left(\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2-1}}$		
Paired testing			
$\mu_d = d_0$	$T = \dfrac{\bar{d}-d_0}{S_d/\sqrt{n}}$	$\mu_d \neq d_0$ $\mu_d > d_0$ $\mu_d < d_0$	$T < -t_{\alpha/2}$ and T> $t_{\alpha/2}$ $T > t_\alpha$ $T < -t_\alpha$

Hypothesis Testing for Proportion

This hypothesis aims to determine the difference in the proportion of one population with a statement of a certain proportion, or the difference in the proportions of one population to another population. A summary of hypothesis testing for proportion is as follows:

H_0	Statistical Test	H_1	Critical region
One Population			
$p = p_0$	$Z = \dfrac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$p \neq p_0$ $p > p_0$ $p < p_0$	$Z < -z_{\alpha/2}$ and $Z > z_{\alpha/2}$ $Z > z_\alpha$ $Z < -z_\alpha$
Two Populations			
$p_1 = p_2$	$Z = \dfrac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ $\hat{p} = \dfrac{x_1+x_2}{n_1+n_2}$	$p_1 \neq p_2$ $p_1 > p_2$ $p_1 < p_2$	$Z < -z_{\alpha/2}$ and $Z > z_{\alpha/2}$ $Z > z_\alpha$ $Z < -z_\alpha$

Hypothesis Testing for Variance

A summary of hypothesis testing for variance is:

H_0	Statistical Test	H_1	Critical region
One Population			
$\sigma^2 = \sigma_0^2$	$\chi^2 = \dfrac{(n-1)S^2}{\sigma_0^2}$	$\sigma^2 \neq \sigma_0^2$ $\sigma^2 > \sigma_0^2$ $\sigma^2 < \sigma_0^2$	$\chi^2 < {\chi^2}_{1-\alpha/2,(n-1)}$ and $\chi^2 > {\chi^2}_{\alpha/2,(n-1)}$ $\chi^2 > {\chi^2}_{\alpha,(n-1)}$ $\chi^2 < {\chi^2}_{1-\alpha,(n-1)}$
Two Population			
$\sigma_1^2 = \sigma_2^2$	$F = \dfrac{S_1^2}{S_2^2}$	$\sigma_1^2 \neq \sigma_2^2$ $\sigma_1^2 > \sigma_2^2$ $\sigma_1^2 < \sigma_2^2$	$F < f_{1-\alpha/2,(n_1-1),(n_2-1)}$ and $F > f_{\alpha/2,(n_1-1),(n_2-1)}$ $F > f_{\alpha,(n_1-1),(n_2-1)}$ $F < f_{1-\alpha,(n_1-1),(n_2-1)}$

Exercises

1. Aircrew escape systems are powered by a solid propellant. The burning rate of this propellant is an important product characteristic. Specifications require that the mean burning rate must be 50 cm/s. Noted that the standard deviation of burning rate is $\sigma = 2$ cm/s. The experimenter decides to specify a type I error probability or significance level of α = 0.05 and selects a random sample of n = 25 and obtains a sample average burning rate of $\bar{x} = 51.3$ cm/s. What conclusions should be drawn?

   The answer:

   $\mu_0 = 50$ cm/s $\quad\quad \sigma = 2$ cm/s

   $n = 25 \quad\quad \bar{x} = 50$ cm/s

   The Hypothesis:

   $H_0: \mu = \mu_0$

   $H_1: \mu \neq \mu_0$

   $\alpha = 10\%$

   Rejection Criteria:

   Reject H_0 if $Z < -z_{0.025}$ and $Z > z_{0.025}$

   Statistical test:

   $$Z = \frac{51.3 - 50}{\frac{2}{\sqrt{25}}}$$

   $Z = 3.25$

   The Result:

   $Z > z_{0.025} \rightarrow 3.25 > 1.96$

   Conclusion:

   Because the $Z > z_{0.025}$, we reject H_0 at the 0.05 significance level.

   We can conclude that the mean burning rate differs from 50 cm/s, based on a sample of 25 measurements. In fact, there is strong evidence that the mean burning rate exceeds 50 cm/s.

2. The average registration time for students from the previous semester was 45 minutes. In this semester a new system was created after testing using 15 student accounts, it

is known that the average registration time is 42 minutes with a standard deviation of 10 minutes. With the new system, whether the current mean is less than 45 minutes? (use α = 5%)

$\mu_0 = 45$ $\bar{x} = 42$

$n = 15$ $s = 10$

The hypothesis:

$H_0: \mu = \mu_0$

$H_1: \mu < \mu_0$

$\alpha = 5\%$

Rejection Criteria:

Reject H_0 if $T < -t_{0.05}$

Statistical test:

$$T = \frac{42 - 45}{\frac{10}{\sqrt{15}}}$$

$T = -1.116$

The result:

$T > -t_{0.05} \rightarrow -1.116 > -1.796$

Conclusion:

Because the $T > -t_{0.05}$, we fail to reject H_0 at the 0.05 significance level.
We conclude that the average registration time of the new system and the old system is not different, although the value is different (still in the same range or region).

3. An automated filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $S^2 = 0.0153$. If the variance of fill volume exceeds 0.01, an unacceptable proportion of bottles will be underfilled or overfilled. Is there evidence in the sample data to suggest that the manufacturer has a problem with underfilled or overfilled bottles? Use α = 0.05, and assume that fill volume has a normal distribution.

$S^2 = 0.0153$

$n = 20$ $\sigma_0^2 = 0.01$

The Hypothesis

$H_0: \sigma^2 = \sigma_0^2$

$H_1: \sigma^2 > \sigma_0^2$

$\alpha = 5\%$

Rejection Criteria:

Reject H_0 if $\chi^2 > {\chi^2}_{\alpha,(n-1)}$

Statistical Test:

$$\chi^2 = \frac{(19)0.0153}{0.01}$$

$\chi^2 = 29.07$

The Result:

$\chi^2 < {\chi^2}_{0.05,(19)} \rightarrow 29.07 < 30.14$

Conclusion:

Because the $\chi^2 < {\chi^2}_{0.05,(19)}$, we fail to reject H_0 at the 0.05 significance level.

We conclude that there is no strong evidence that the variance of fill volume exceeds 0.01. So there is no strong evidence of a problem with incorrectly filled bottles.